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Grant 664782

Nonlinear Gamow Vectors in nonlocal optical propagation

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101° Congresso della Società Italiana di Fisica, Rome December 21-25 2015, the 21th June 2015

Subject

Dispersive Shock Waves

Challenge

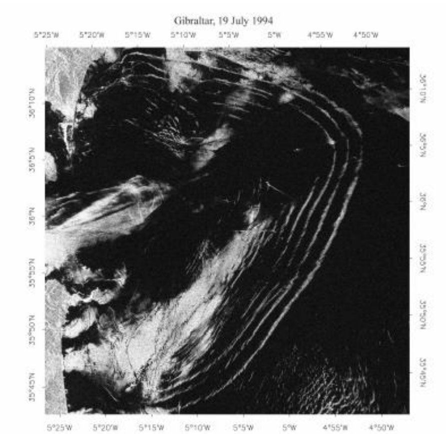
Description of Shock Waves beyond Shock Point

Outline

- Shock phenomena
- Reverted Harmonic Oscillator
- Numerical Simulations
- Experimental Results

Dispersive Shock Waves in Physics

Shock in fluidodynamics



Shock in nonlinear optics

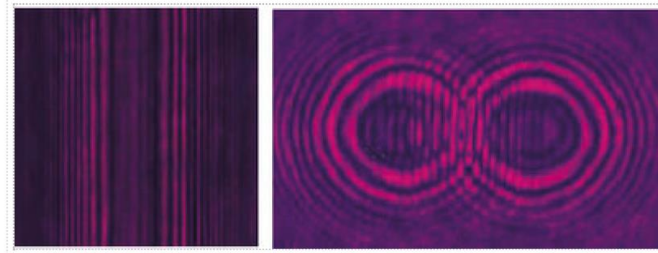


Figure 3: Experimental images of two one-dimensional (left) and two interacting two-dimensional (right) optical DSWs performed at Princeton University (Wan, Jia, Fleischer 2007).

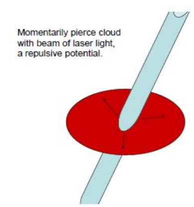
Shock in supernova explosion

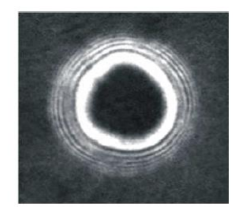
Illustration of propagation W44 shock waves in the molecular cloud. Keio University/NAOJ

Shock in supersonic flows

Shock in Bose-Einstein condensation

http://jilawww.colorado.edu/bec/papers.html





- Radially expanding dispersive shock wave

M.A. Hoefer, M.J. Ablowitz, I. Coddington, E.A. Cornell, P. Engels, and V. Schweikhard, Phys. Rev. A **74,** 023623 (2006)

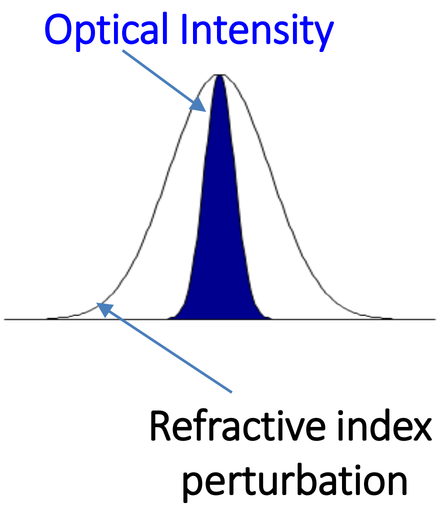
Nonlinear Schrödinger Equation

Local NLS equation

$$i\frac{\partial\psi}{\partial z}+\frac{1}{2}\frac{\partial^2\psi}{\partial x^2}-P|\psi(x)|^2\psi=0$$

Nonlocal NLS equation

$$i\frac{\partial\psi}{\partial z}+\frac{1}{2}\frac{\partial^2\psi}{\partial x^2}-P\int K(x-x')|\psi(x')|^2dx'\,\psi=0$$



$$i\frac{\partial\psi}{\partial z}+\frac{1}{2}\frac{\partial^2\psi}{\partial x^2}-PK(x)*|\psi(x)|^2\,\psi=0$$

From NLS to Hydrodynamic Model

$$\psi \to \varepsilon^{-2}\psi$$
$$\partial_x \to \varepsilon^{-1}\partial_x$$
$$\partial_z \to \varepsilon^{-3}\partial_z$$

$$i\frac{\partial\psi}{\partial z} + \frac{1}{2}\frac{\partial^2\psi}{\partial x^2} - |\psi(x)|^2\psi = 0 \longrightarrow i\varepsilon\frac{\partial\psi}{\partial z} + \frac{\varepsilon^2}{2}\frac{\partial^2\psi}{\partial x^2} - |\psi(x)|^2\psi = 0$$

$$\varepsilon = \sqrt{\frac{L_{nl}}{L_d}}$$

WKB Approach

If $\psi(z,x) = \sqrt{\varrho(z,x)}\, e^{i\phi/\varepsilon}$ and $u = \partial_x\phi$

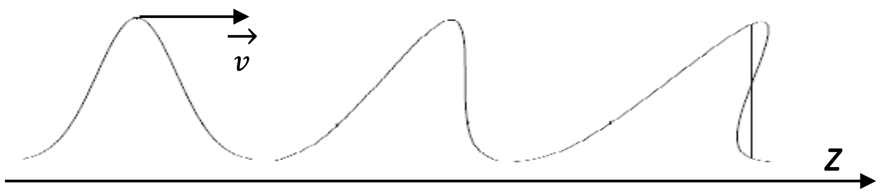
$$i\varepsilon\frac{\partial\psi}{\partial z} + \frac{\varepsilon^2}{2}\frac{\partial^2\psi}{\partial x^2} - |\psi(x)|^2\psi = 0 \longrightarrow$$

$u_z + uu_x + \varrho_x = 0$	Continuity eq.	0° order
$\varrho_z + (\varrho u)_x = 0$	Eulero eq.	1° order

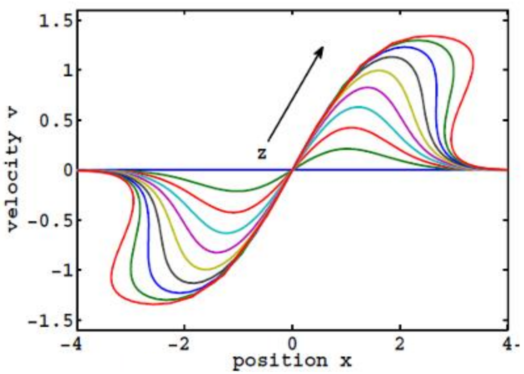
Dynamics driven by the phase tilt, the intensity follows

Hopf Equation

$$u_z + u u_x = 0$$

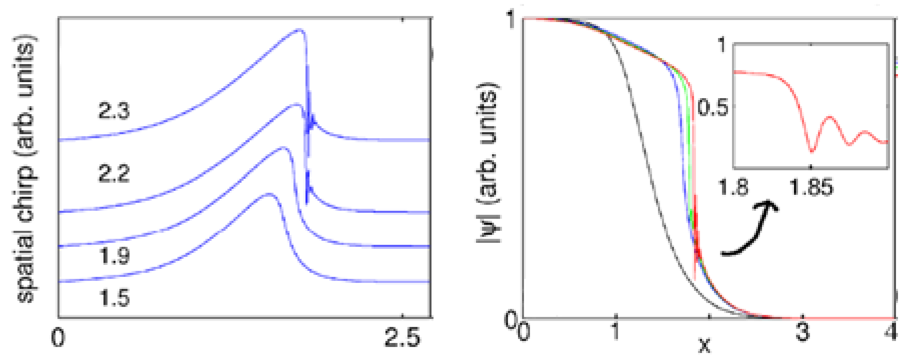


Regularization by dissipation

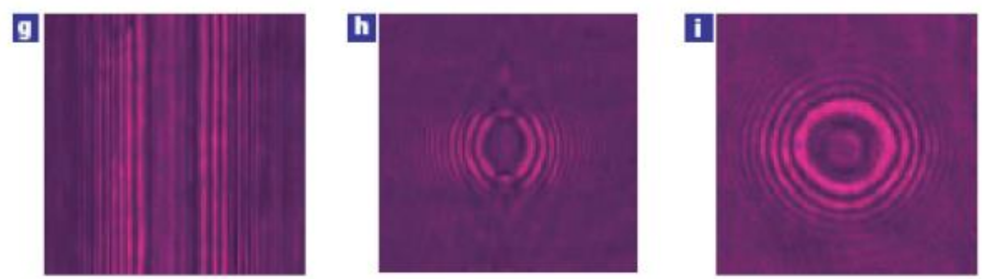


N. Ghofraniha, S. Gentilini, V. Folli, E. DelRe, and C. Conti PRL ***109****, 243902, 2012*

Regularization by dispersion



N. Ghofraniha, C. Conti, G. Ruocco, S. Trillo, PRL 99, 043903, 2007



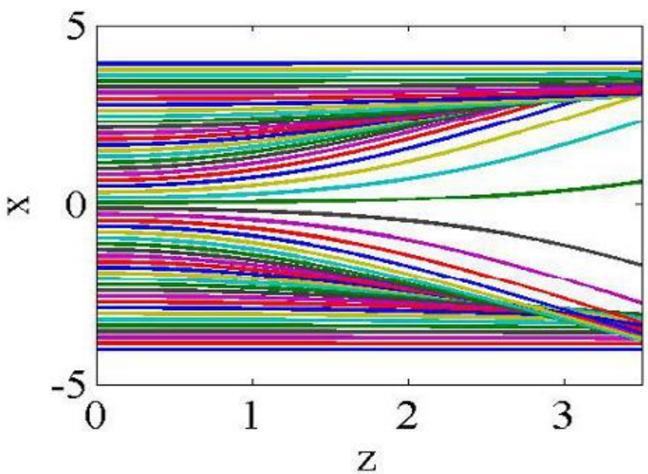
W. Wan, S. Jia, J. W. Fleischer, Nature Phys., 2006

Characteristic lines Method & Shock Point

N. Ghofraniha, S. Gentilini, V. Folli, E. DelRe, and C. Conti PRL ***109****, 243902, 2012*

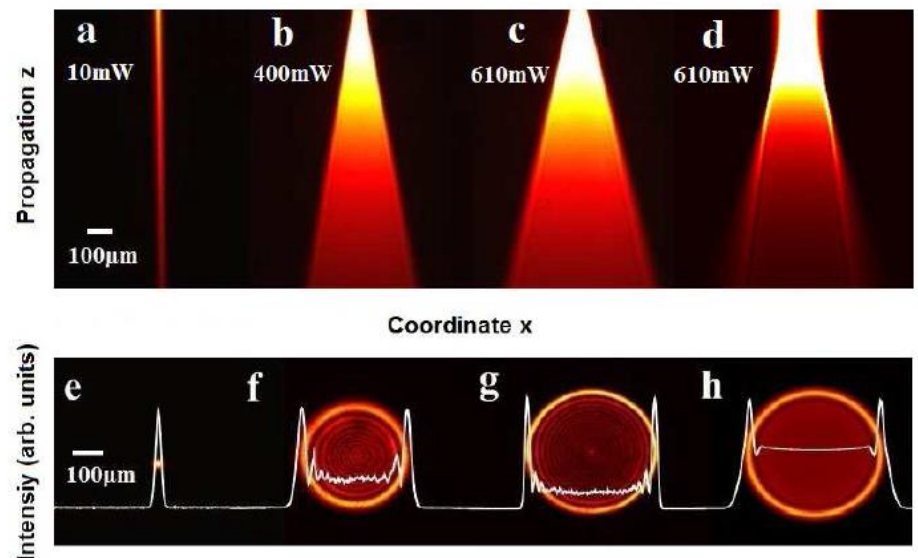
$$u_z + uu_x = 0$$

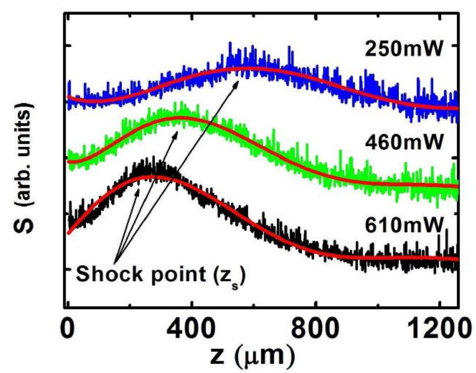
$$\begin{cases} \dfrac{du}{dz} = 0 \\ \dfrac{dx}{dz} = u \end{cases}$$

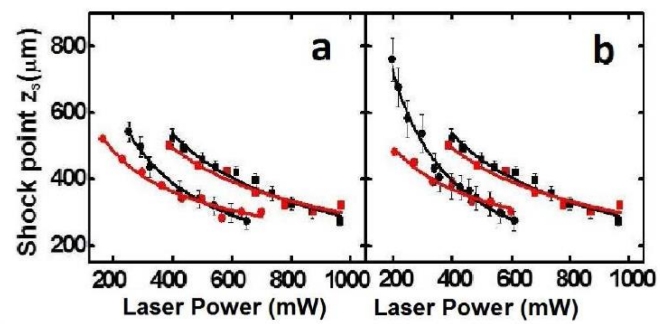


The Hydrodynamical regime
is only valid before the shock point

The characteristic lines method allows
to predict the scaling law of the shock point



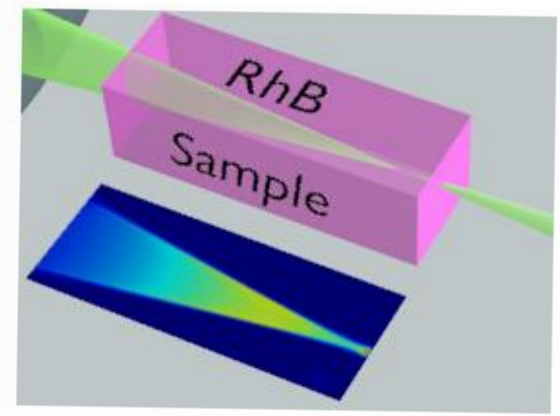




Challenge!

the description of shock waves beyond the shock point

Nonlocal NLS Equation



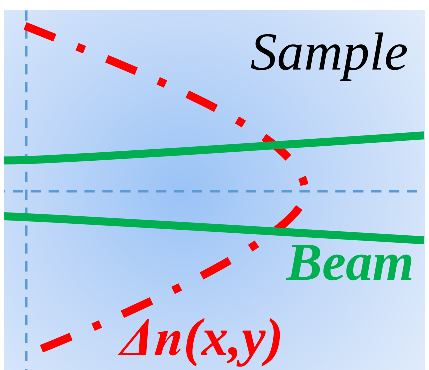
$$i\frac{\partial\psi}{\partial z}+\frac{1}{2}\frac{\partial^2\psi}{\partial x^2}-PK(x)*|\psi(x)|^2\,\psi=0$$

Highly Nonlocal Approximation

$$\kappa(x)\approx K(x)*|\psi(x)|^2 \longrightarrow i\frac{\partial\psi}{\partial z}+\frac{1}{2}\frac{\partial^2\psi}{\partial x^2}-P\kappa(x)\psi=0$$

$$\kappa(x)\approx\kappa_0^2-\frac{1}{2}\kappa_2^2x^2$$

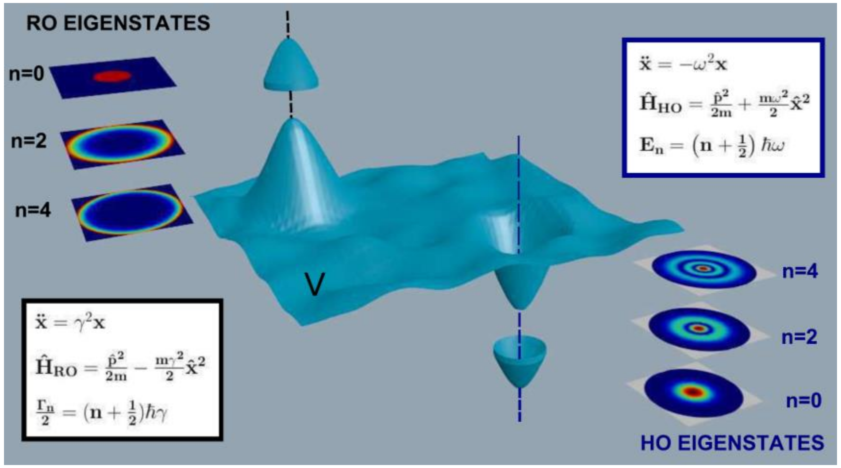
Snyder, A. W. & Mitchell, D. J.
Accessible Solitons,
Science 276, 1538–1541, 1997



Linear Schrödinger Equation for Nonlinear Propagation

$$i\frac{\partial\psi}{\partial z}=\hat{H}\psi \longrightarrow \hat{H}=\frac{1}{2}\hat{p}^2+V(x) \longrightarrow \begin{array}{l}\hat{p}=-i\partial_x\\ V(x)=P\kappa(x)\end{array}$$

Reversed Harmonic Oscillator

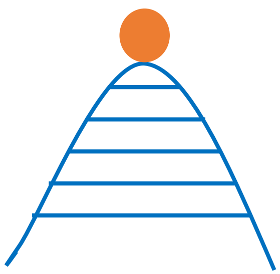


Physical realization of the Glauber quantum oscillator, S. Gentilini, M.C. Braidotti, G. Marcucci, E. DelRe, Claudio Conti, submitted

Reversed Harmonic Oscillator

$$\hat{H} = \frac{1}{2}\hat{p}^2 - \frac{1}{2}\gamma^2 x^2$$

$$\gamma^2 = P\kappa_2^2$$



Glauber ⟶ *Amplifiers, Attenuators, and Schrödinger's Cat, Annals of the New York Academy of Sciences 480, 336–372, 1986*

A. Bohm ⟶ Irriversible Quantum Mechanics

Bohm, A. R. Time Asymmetric Quantum Physics Phys.Rev. A 60, 861–876, 1999

From HO to RO

Complex extension

At any eigenstate of the HO we can associate two solutions of the RO

$$\hat{H}_{ho} = \frac{1}{2}\hat{p}^2 + \frac{1}{2}\omega^2 x^2 \xrightarrow{x \to \sqrt{\mp i}x} \hat{H}_{ro} = \frac{1}{2}\hat{p}^2 - \frac{1}{2}\gamma^2 x^2$$

$\phi_0(x) = e^{-x^2}$ Ground state of the Harmonic oscillator

$\mathfrak{f}_0^{\pm}(x) = e^{\mp i x^2}$ Analytical prolongation

$E_0 \to E_0^{\pm} = \pm i E_0$ Eigenvalues of the RO with complex energy !

Gamow vector Ground state (shock front) NOT NORMALIZABLE!!!!

Gamow Vectors

RO eigenfunctions

$$f_n^{\pm}(x) = \frac{\sqrt[4]{\pm i\gamma}}{\sqrt{2^n n! \sqrt{\pi}}} H_n(\sqrt{\pm i\gamma}\, x) e^{\mp i\frac{\gamma}{2}x^2}$$

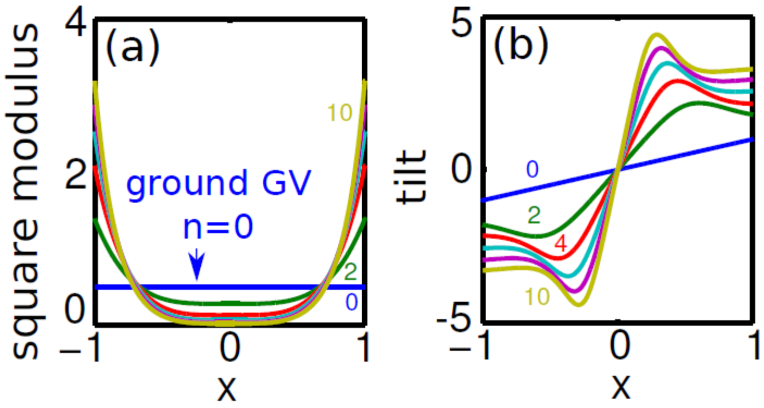


Fig.: (Color online) (a) GV $|f_n^-(x)|^2$ for increasing even order *n;* (b) $\partial_x \arg[f_n^-(x)]$ for increasing even order;

GVs are numerable generalized eigenvectors of $\hat{H}$ with complex eigenvalues. They exist in the Rigged Hilbert space (RHS), where they furnish a generalized basis for normalizable wavepackets:

Quantized decay rate!

$$\psi^G(x,z) = \sum_{n=0}^{\infty} \phi_n^G(x)\ e^{-iE_n^R z - \frac{\Gamma_n}{2} z} \quad \text{with} \quad E_n = E_n^R - i\Gamma_n/2 \quad \Gamma_n = \gamma(1+2n)$$

GV have exponential evolution.

Numerical Validation

We compare simulation with the solution of the nonlocal Schrödinger equation (in the finite nonlocality case)

Numerical Simulation vs Theory

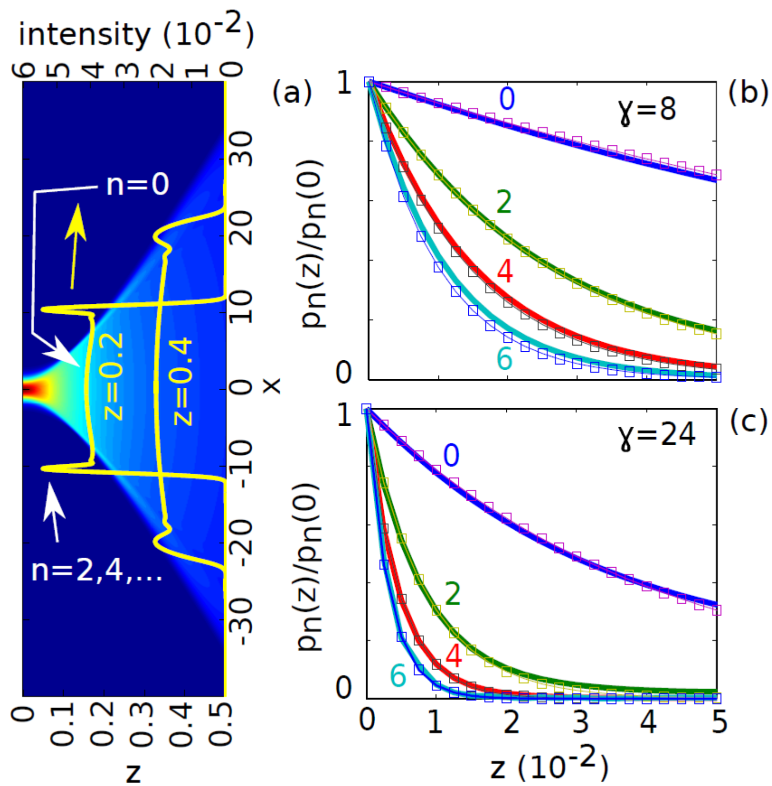
Propagated equation

$$i\frac{\partial\psi}{\partial z}+\frac{1}{2}\frac{\partial^2\psi}{\partial x^2}-PK(x)*|\psi(x)|^2\,\psi=0$$

Gaussian wave packet

$$\psi(x,0)=\frac{e^{-x^2/2}}{\sqrt[4]{\pi}}$$

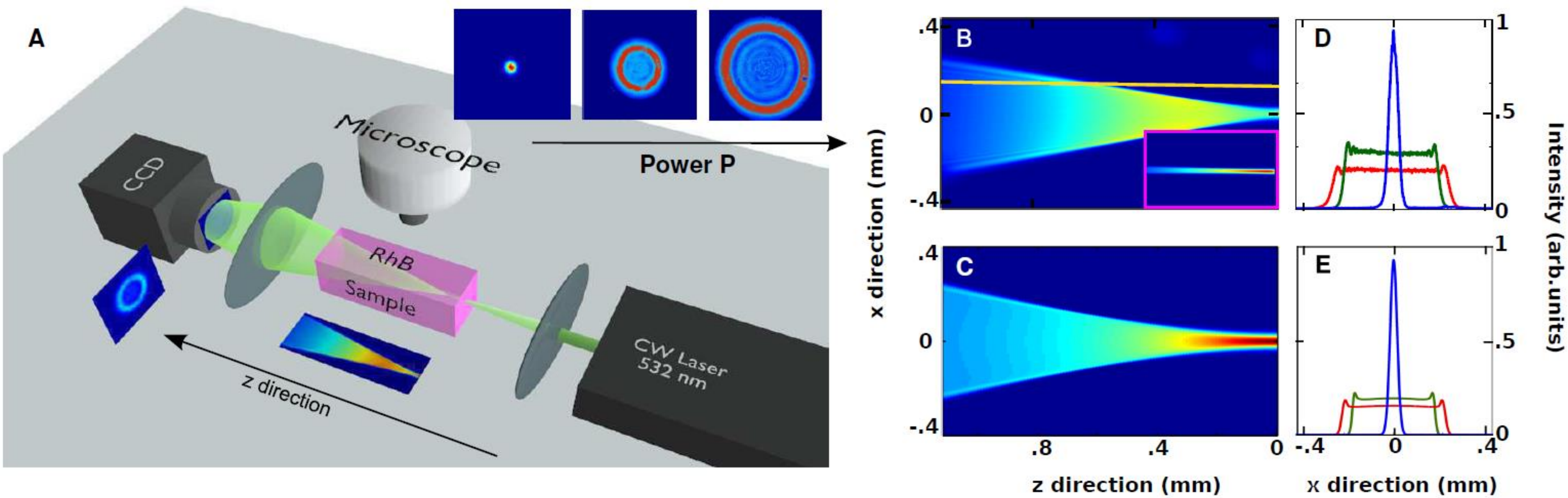
$$p_n(z)=\Gamma_n|\langle f_n^+|\psi(x,0)\rangle|^2e^{-\Gamma_n z}$$



The evolution AFTER the shock point is described by the superposition of Gamow vectors!!!

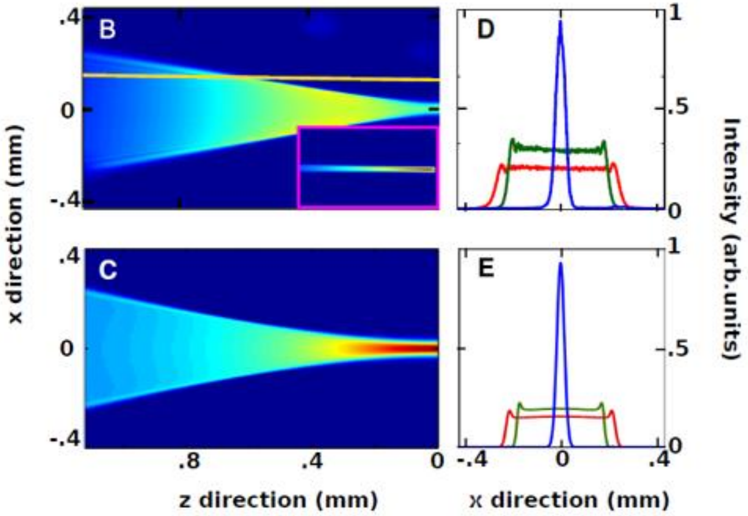
Experimental Results

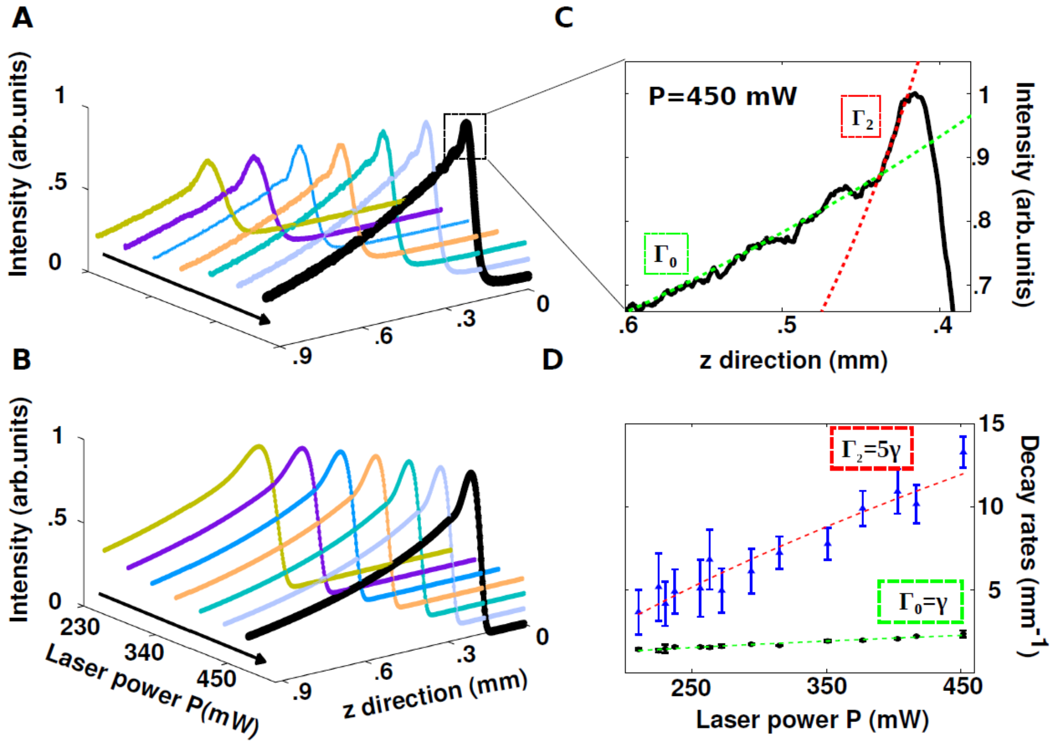
Experimental Set-Up



A. Schematics of experimental setup to obtain transmitted and top flourescence images of DSWs excited by focusing a cw laser in aqueous solution of Rhodamine B; **B.** Top fluorescence image of the propagating laser at P=380mW; **C.** Numerical solution; **D.** Section of experimental intensity profile at different z; **E.** The same of panel (D) obtained from numerical profile in (C).

Decay Rates





Experimental evidence of the quantization of decay times !!!!

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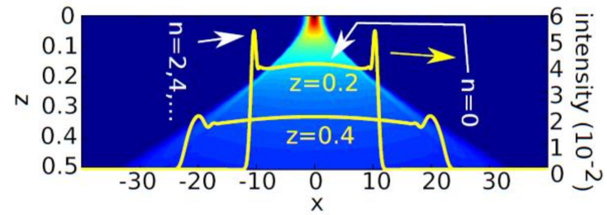
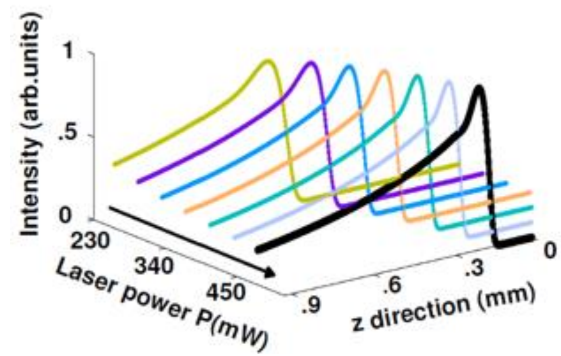
Conclusions

Nonlinear Gamow vector describe shock waves at any z



The quantized decay rates are observed in the experiments and depends on power



Applications

Control of extreme nonlinear regimes (supercontinuum generation)

Analogies of fundamental physical theories

[1] S. Gentilini, M.C. Braidotti, G. Marcucci, E. Del Re and C. Conti, *"Nonlinear Gamow vactors, shock waves and irreversibility in optically nonlocal media"*, Phys. Rev. A **92**, 023801 – Published 3 August 2015

[2] S. Gentilini, M.C. Braidotti, G. Marcucci, E. Del Re and C. Conti, *"Physical realization of the Glauber quantum oscillator"*, submitted;