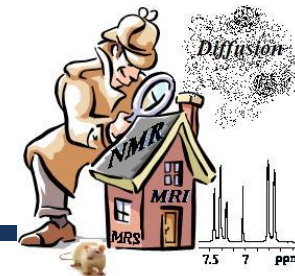




NMR Laboratory



# NMR Anomalous Diffusion Measurements to investigate complex systems: experiments

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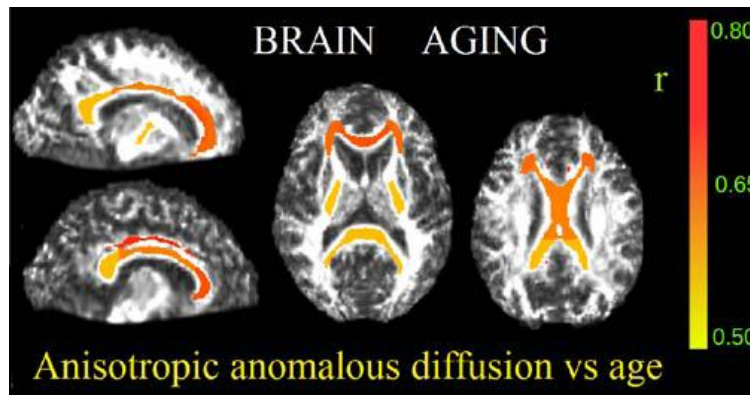
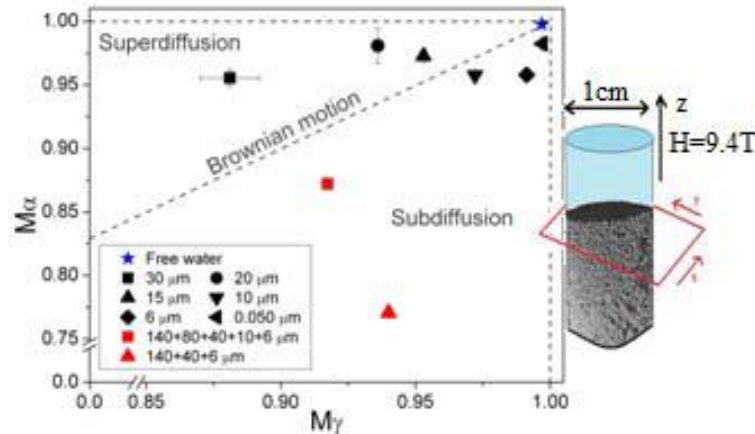
[silvia.capuani@roma1.infn.it](mailto:silvia.capuani@roma1.infn.it)

# NMR Anomalous diffusion Highlights

Fundamental physics

Molecular diffusion in soft condensed matter: porous heterogeneous, complex systems

Experimental corroboration of simulations and theories



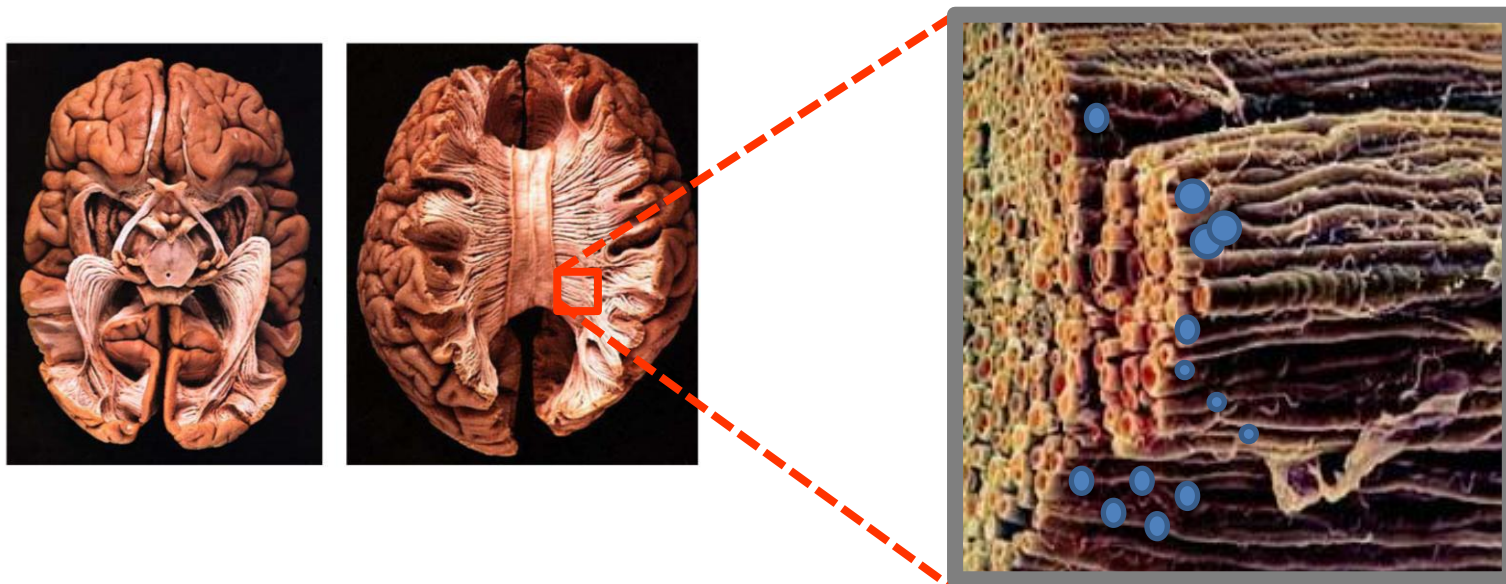
Biophysical applications

Translational longitudinal studies in Humans

Transfer of technology

Clinical application

Why would we want to evaluate the Diffusion parameters of water in materials and biological Tissue?

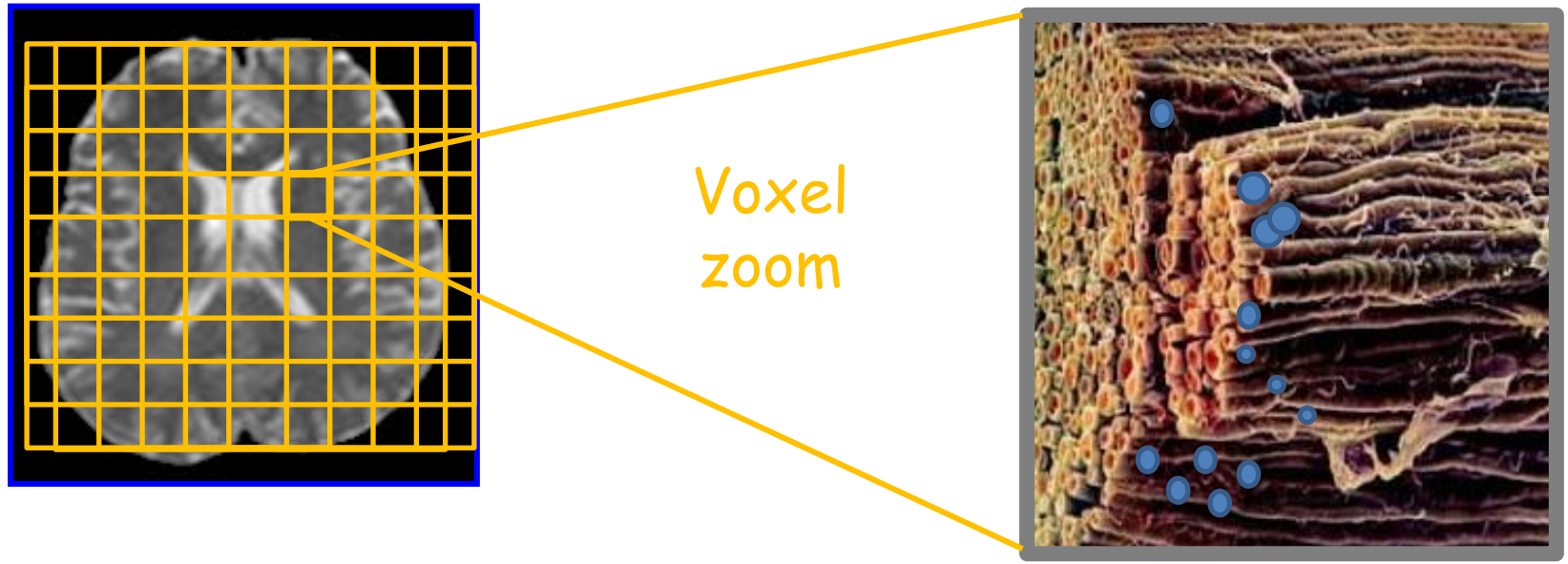


Because Diffusion parameters and diffusion-weighted NMR signal reflect the micro-structural rearrangement of porous materials and tissues

We cannot measure the Diffusion Coefficient  
of water (or of generic ions and molecules)  
using NMR

We can measure the displacement  
of the ensemble of spins in our sample  
and infer the Diffusion Coefficient.

Diffusion MR images can measure water proton displacements at the cellular level



Probing motion at microscopic scale ( $10\ \mu\text{m}$ ), orders of magnitude smaller than macroscopic MR resolution (mm)

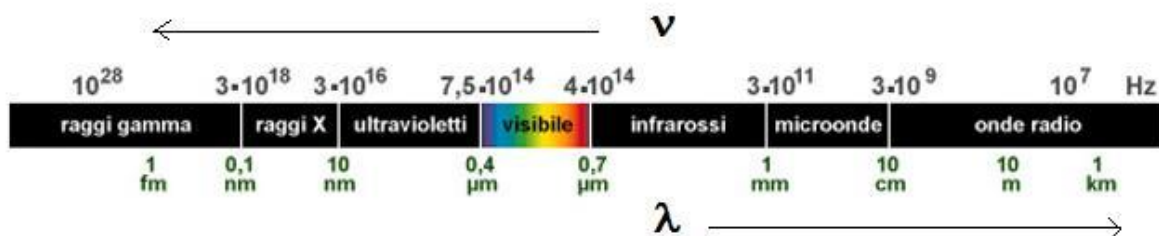
This has found numerous research and clinical applications

Can probe diffusion for time scales:

1 ms to seconds

length scales of displacements:

100 nm to 100 μm



### Conventional diffusion

Porous Media

- Intrinsically multiphase materials
- Pore fluids for NMR detection

Microstructure is important

- Rocks, oil and water reservoirs
- Soils, unconsolidated formation
- Cement and concrete, catalysts
- Food stuff, paper, fabric
- Plant and animal tissues, bone
- Human tissues
- Brain

### Anomalous diffusion

Complex systems

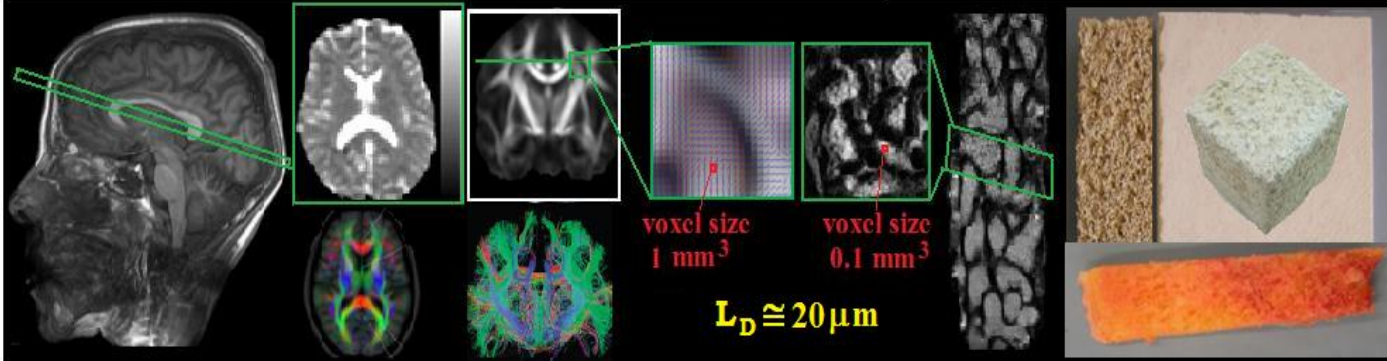


Multiscale

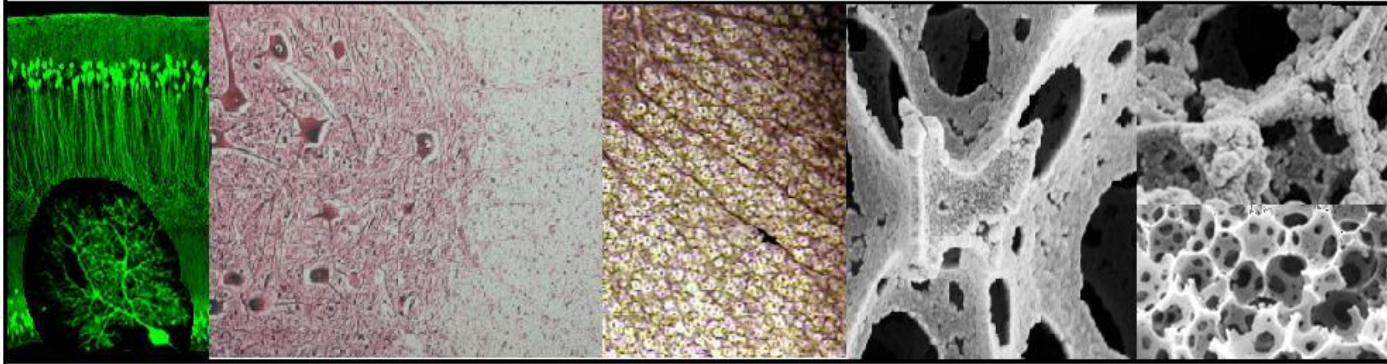
system constituted by structures at very different levels of hierarchical organization



Visible structures with conventional Diffusion Magnetic Resonance



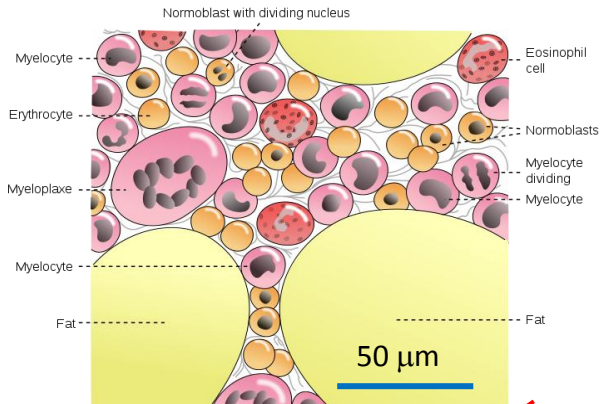
Anomalous diffusion to make the invisible visible



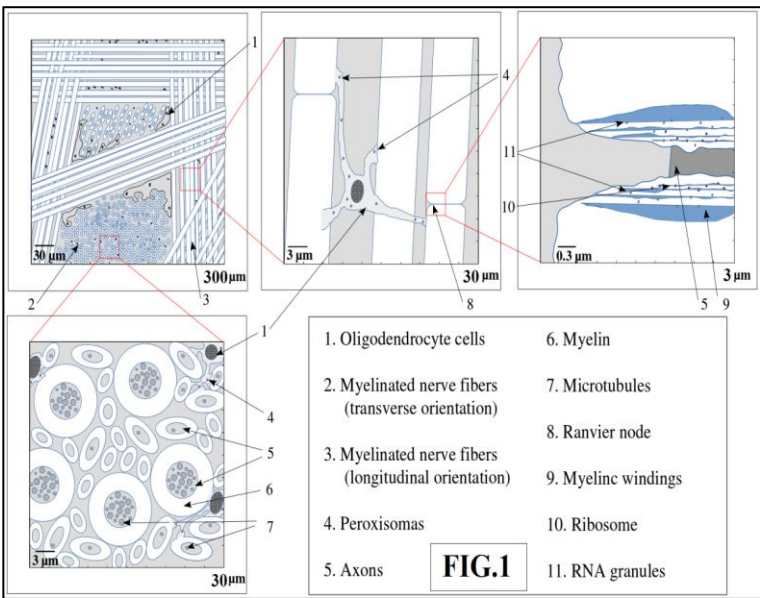
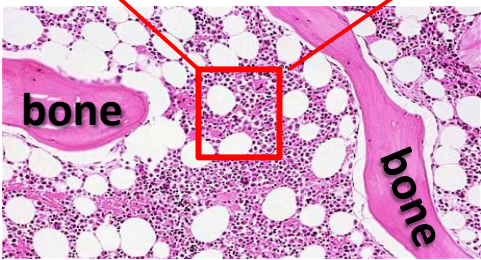
Invisible structures with conventional Diffusion Magnetic Resonance

Gaussian diffusion: diameter of the spatial range, d, accessible to a water molecule in a given diffusion time t

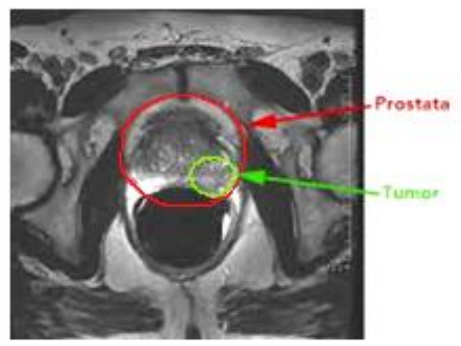
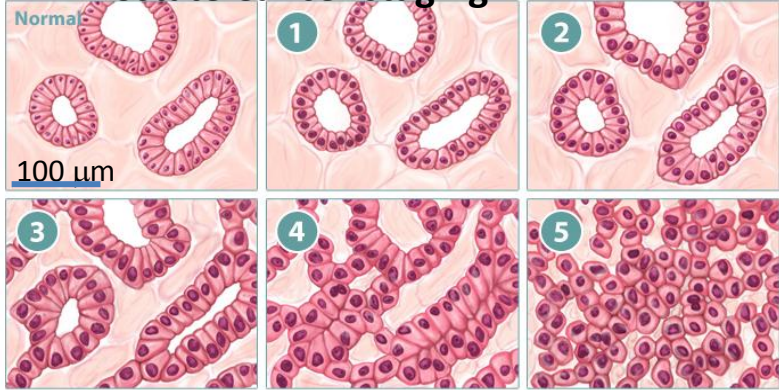
$$L_D = d \cong (6Dt)^{1/2}$$



~~Trabecular bone marrow~~



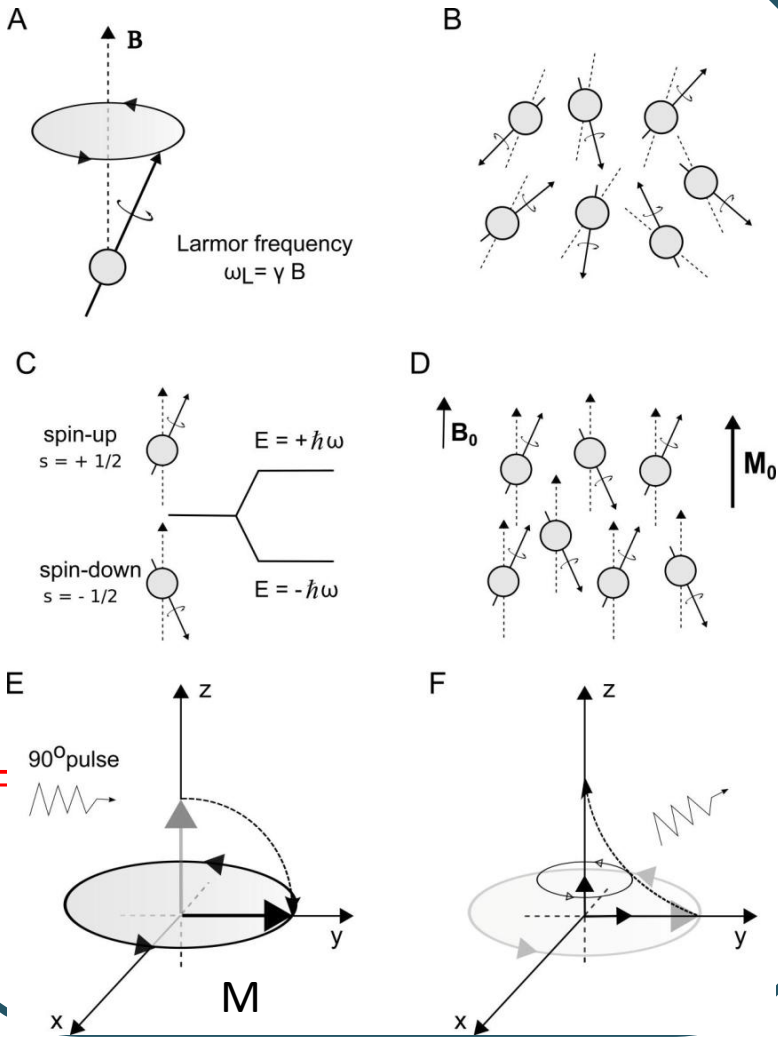
## Prostate Cancer staging





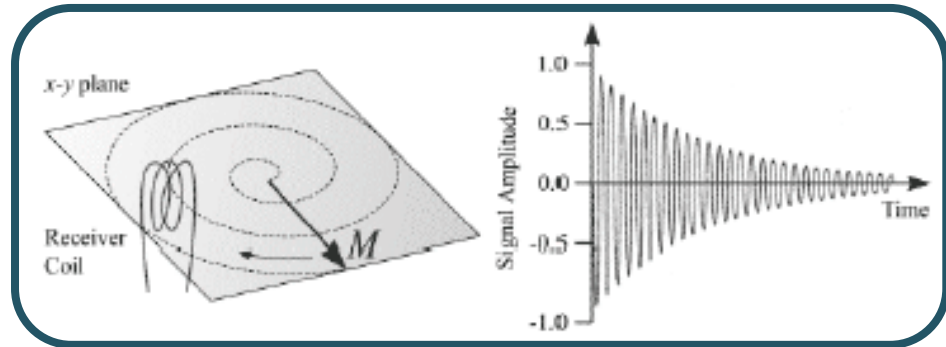
# NMR principles

## Nuclear paramagnetism



NMR signal is an electric signal

f.e.m. induced at the radiofrequency (RF) coil

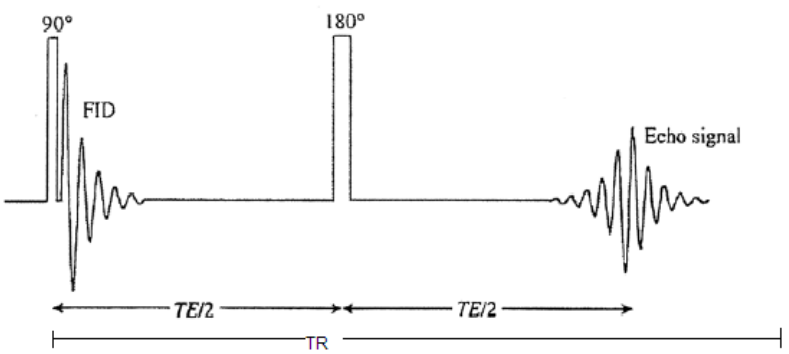
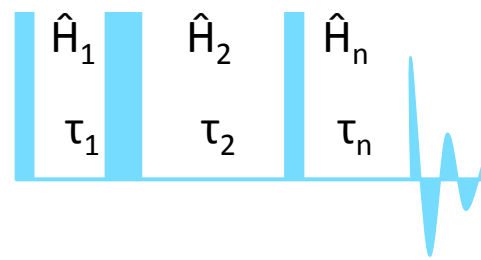


$$S(t) = f(N, T_1, T_2, T_2^*, CS, J, D, \dots)$$

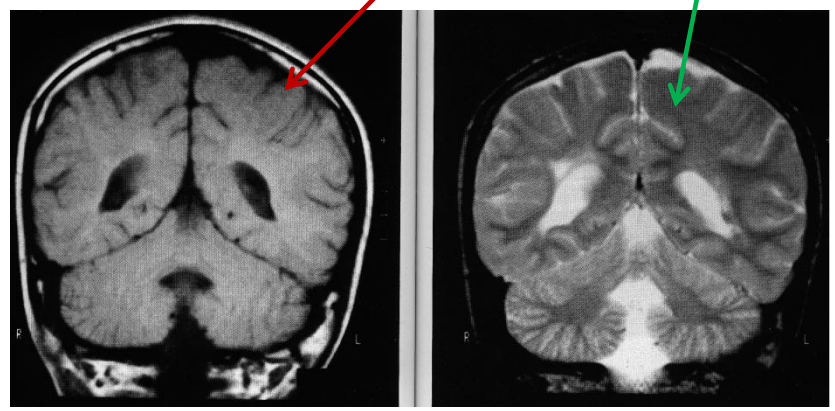
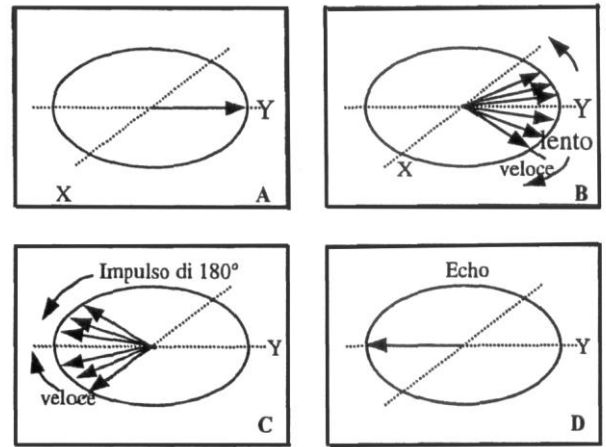
RF pulse sequences

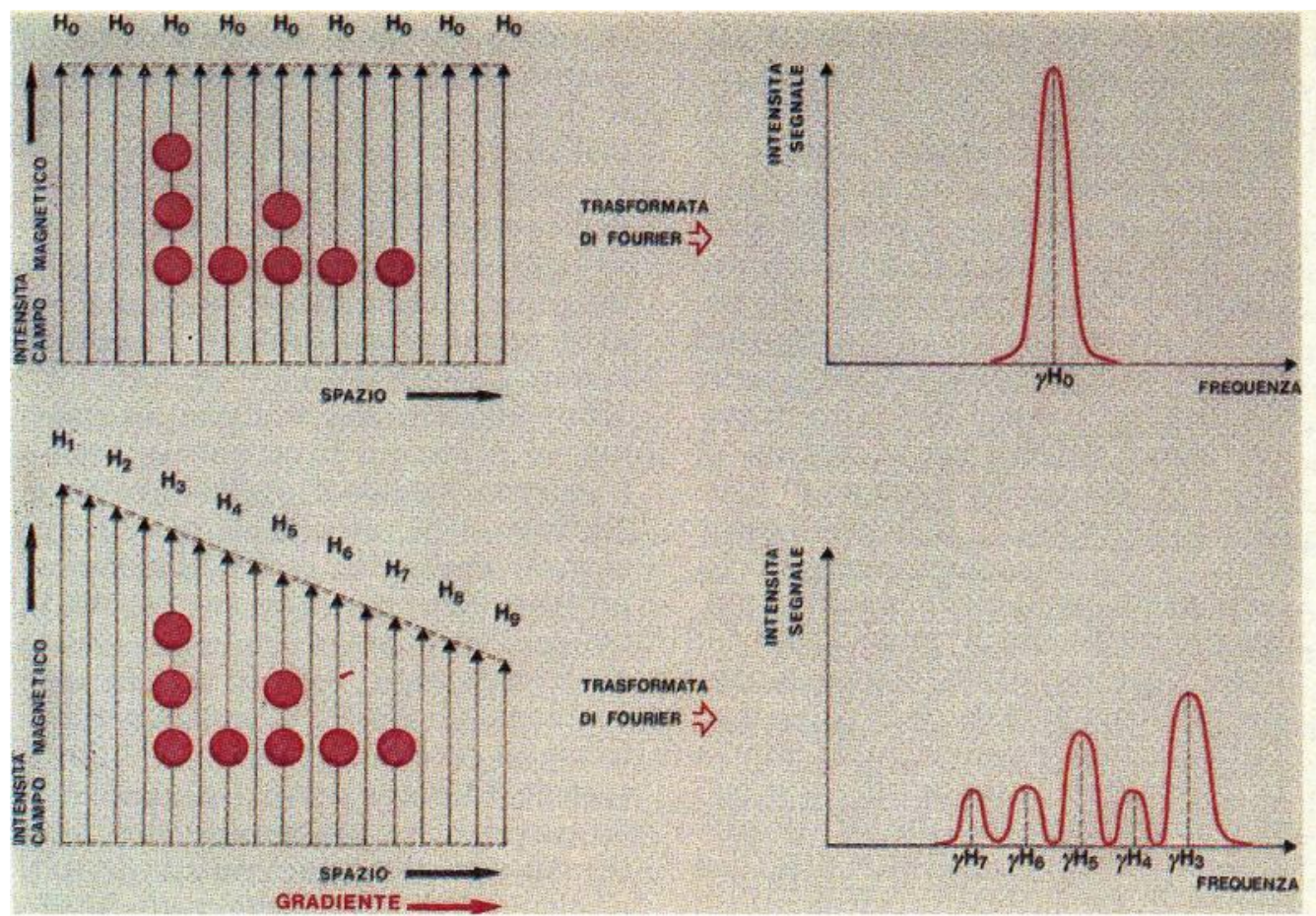
$$S(t) = f(N, T_1, T_2, T_2^*, CS, J, D, \dots)$$

RF sequences



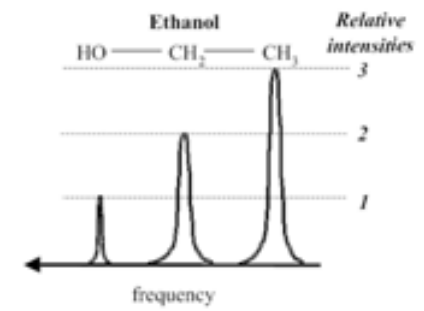
$$S(t) \propto M(T_E) \cong M^0_{xy} (1 - e^{-\frac{T_R}{T_1}}) e^{-\frac{T_E}{T_2}}$$





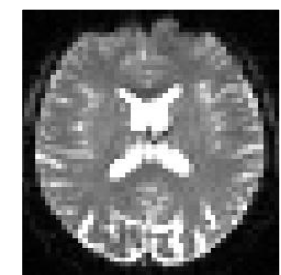
$$\omega_0 = \gamma H_0$$

Spectroscopy

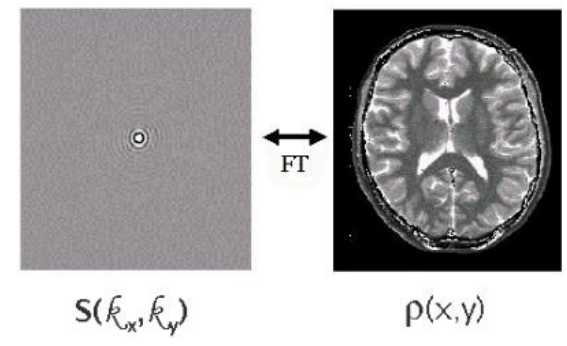
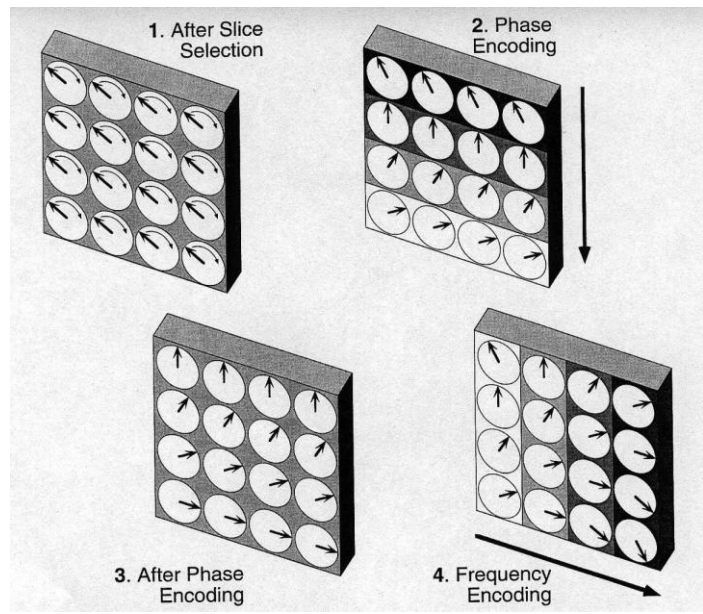
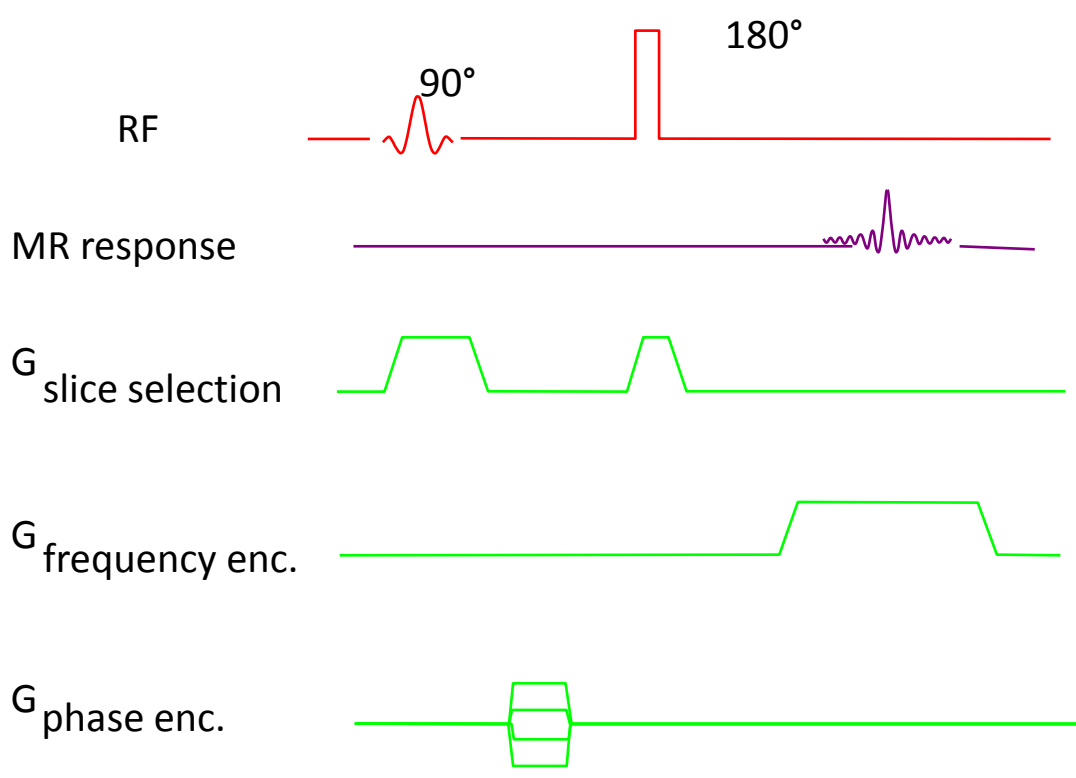


$$\omega(r) = \gamma (H_0 + G \cdot r)$$

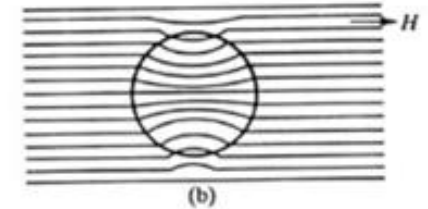
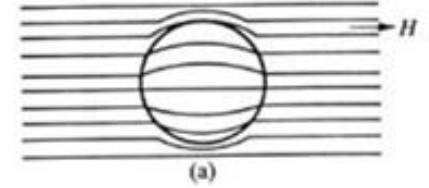
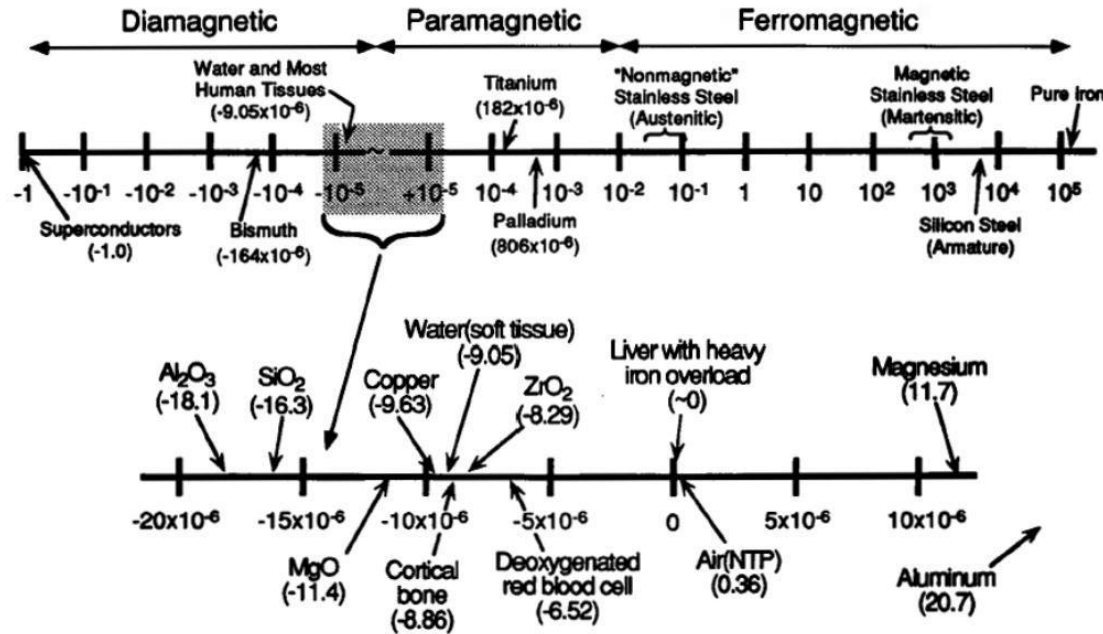
Imaging



$$\omega(\vec{r}) = \omega_0 + \gamma \vec{G} \cdot \vec{r}$$



$$M_0 = \frac{N\gamma^2\hbar^2 I(I+1)}{3KT} H_0 = \chi H_0$$

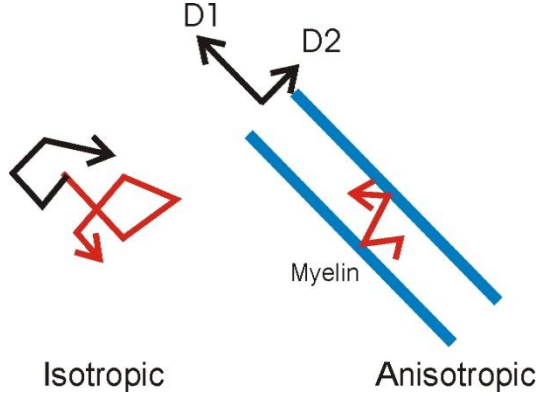
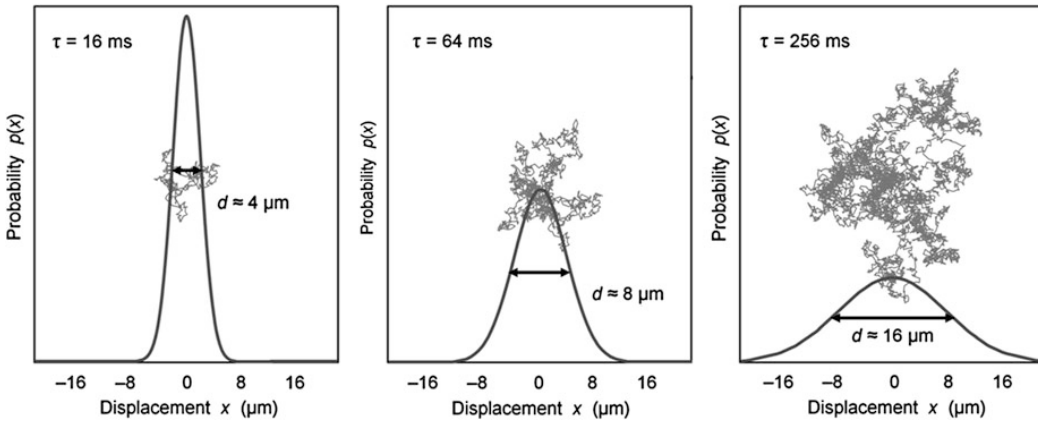
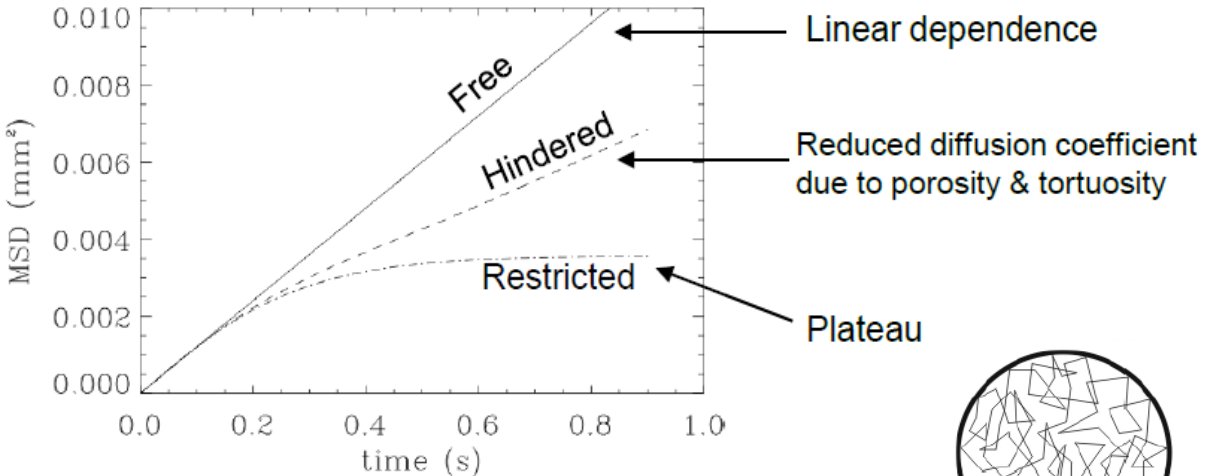


In heterogeneous and complex samples  $\Delta\chi$  at the interface between different tissues or materials

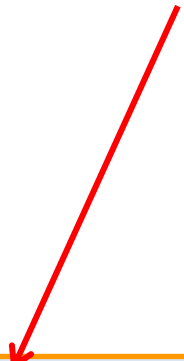
Gaussian Motion Propagator

$$P(\vec{r} | \vec{r}', t) = \frac{1}{(4\pi Dt)^{3/2}} e^{-|\vec{r}-\vec{r}'|^2/4Dt}$$

$$\langle r^2(t) \rangle = 2dDt$$

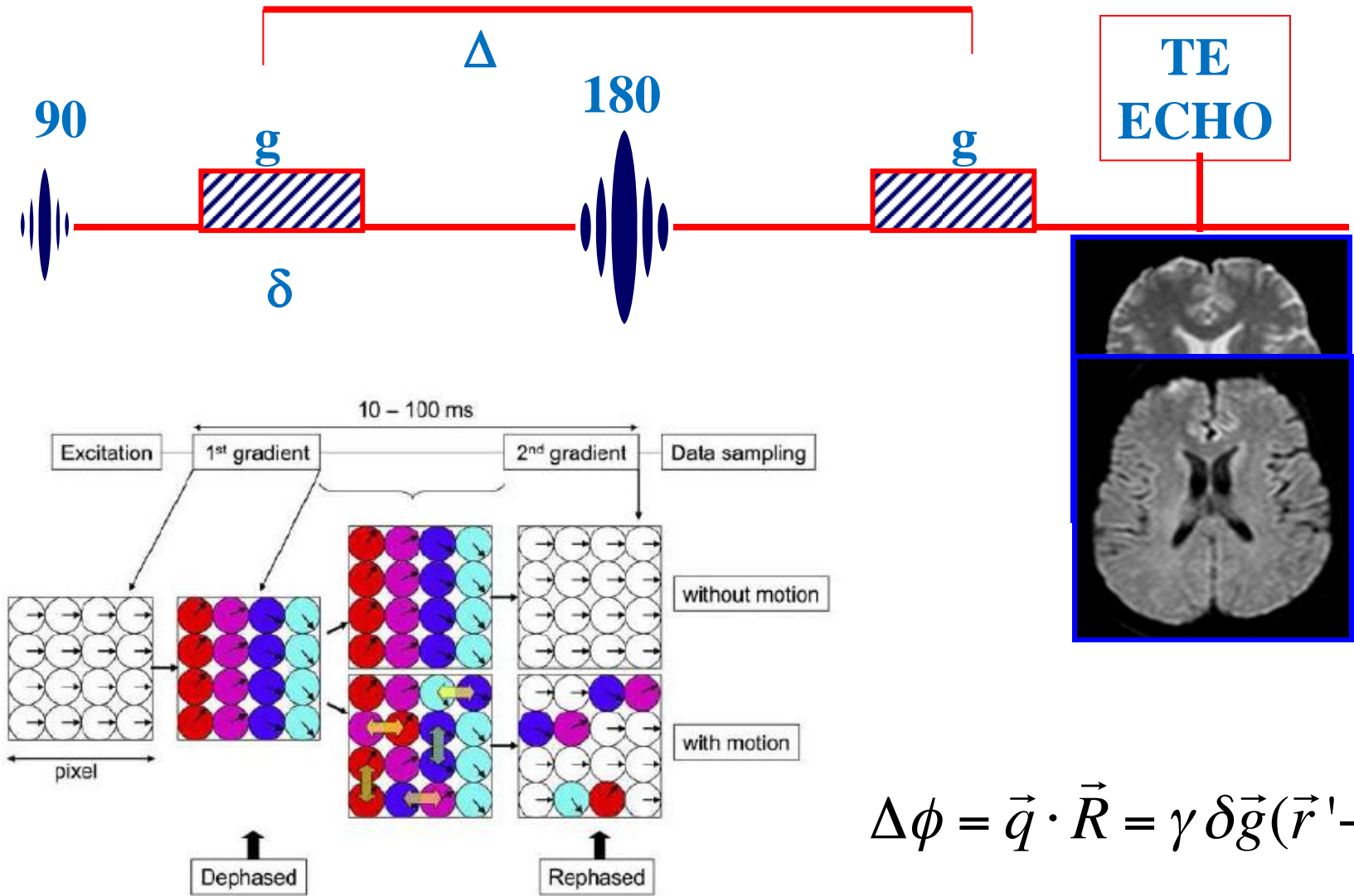


The diffusion weighted signal is proportional to the characteristic function of the diffusion propagator in time  $\Delta$


$$S(\vec{q}, t = \Delta) \propto \int P(\vec{R}, \Delta) e^{i\vec{q} \cdot \vec{R}} \propto W(\vec{q}, t)$$

$$\vec{q} = \gamma \vec{g} \delta$$

$\Delta =$  diffusion time  $t$



$$\Delta\phi = \vec{q} \cdot \vec{R} = \gamma \delta \vec{g} (\vec{r}' - \vec{r})$$



$$b = (\gamma g \delta)^2 \left( \Delta - \frac{\delta}{3} \right)$$

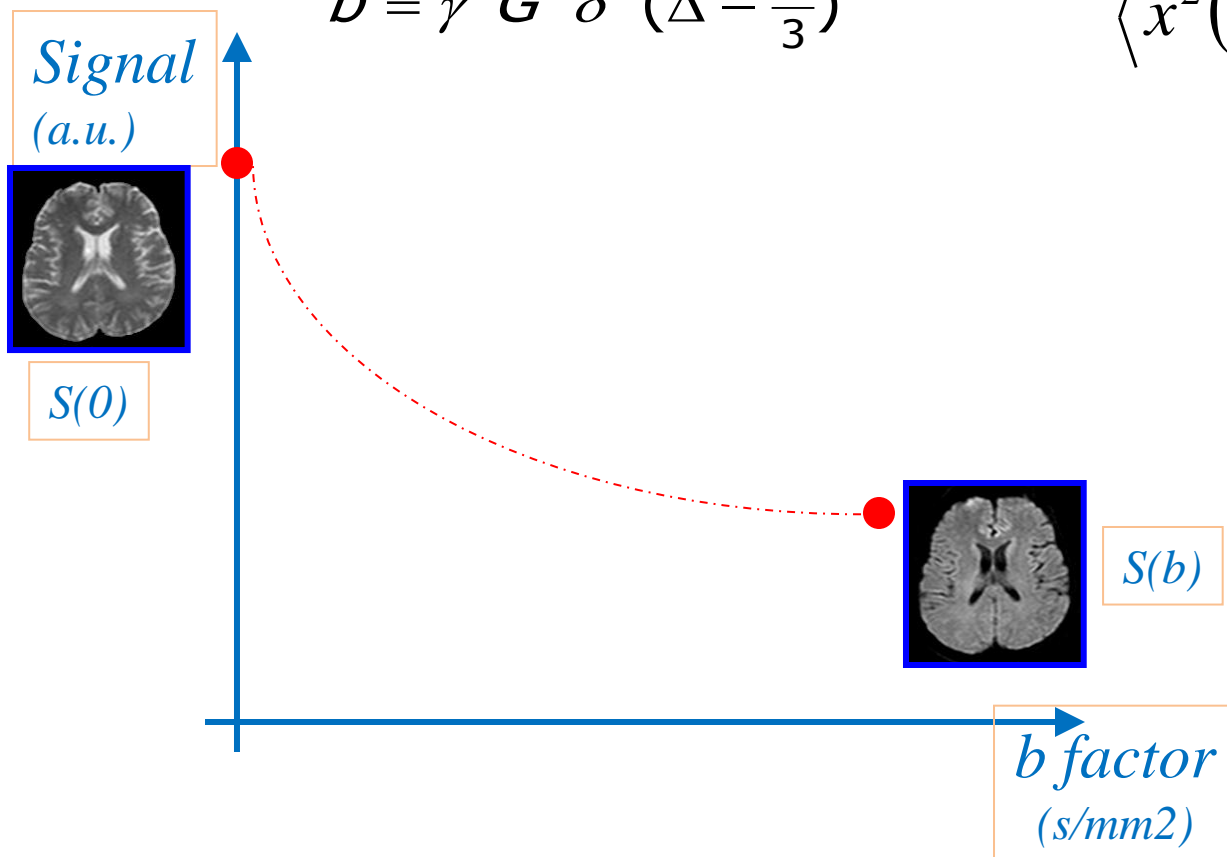


$$S(b) = S(0) \exp(-bD)$$

Gaussian propagator

$$b = \gamma^2 G^2 \delta^2 \left( \Delta - \frac{\delta}{3} \right)$$

$$\langle x^2(t) \rangle \propto Dt$$

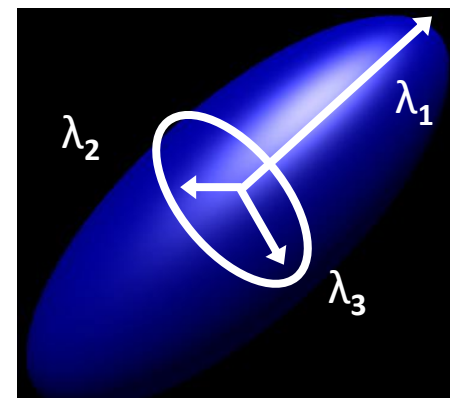


$$\frac{S(t)}{S_0} = \exp\left(-\sum_{i,j=1}^3 b_{ij} D_{ij}\right)$$

$$\underline{D} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{pmatrix}$$

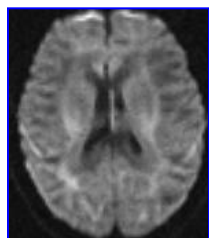
$$\underline{D} = \begin{pmatrix} D_1 & 0 & 0 \\ 0 & D_2 & 0 \\ 0 & 0 & D_3 \end{pmatrix}$$

$v_1, v_2, v_3$

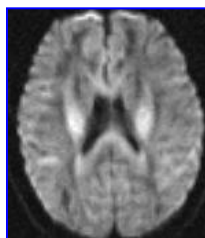


$$D_1 v_1^2 + D_2 v_2^2 + D_3 v_3^2 = c$$

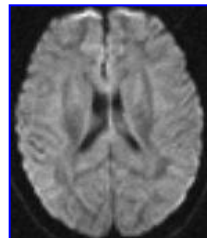
At least six independent parameters are required.  
At least six no-complanar



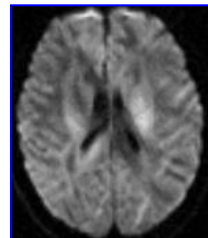
$G_x$   
DWI<sub>x</sub>



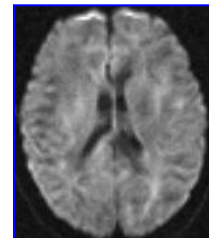
$G_y$   
DWI<sub>y</sub>



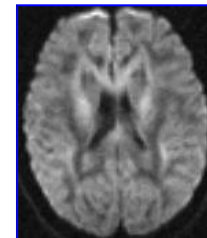
$G_z$   
DWI<sub>z</sub>



$G_{xz}$   
DWI<sub>xz</sub>



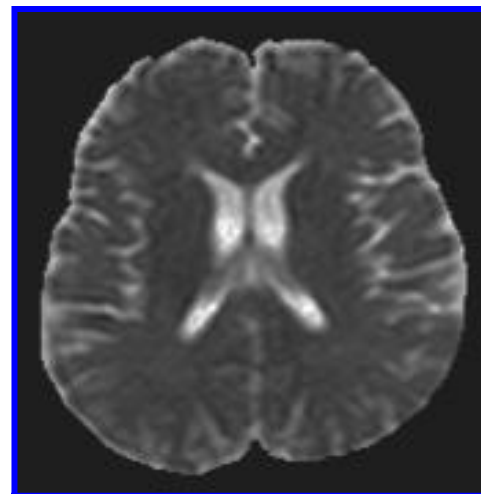
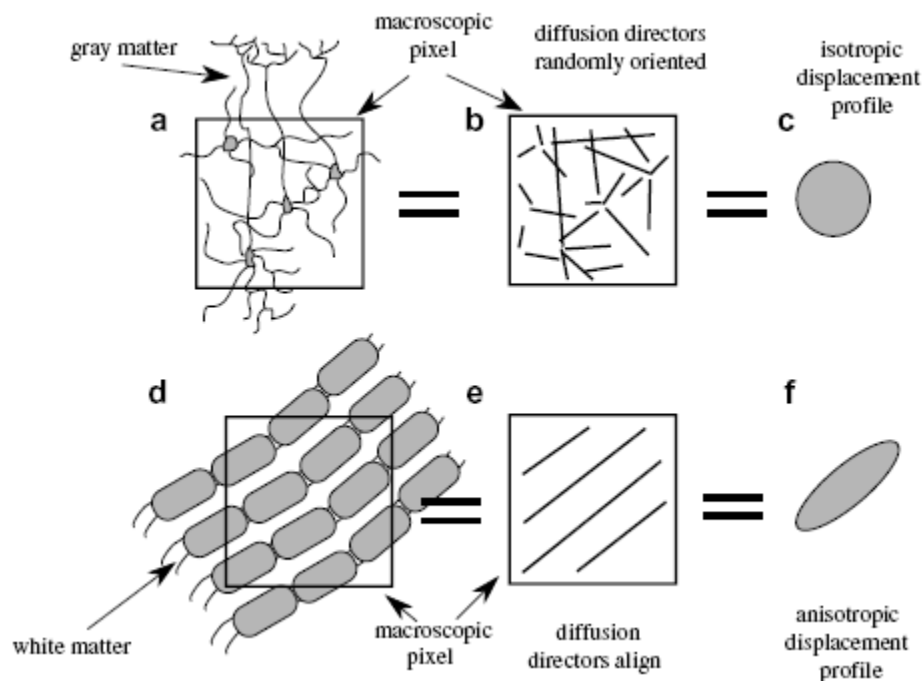
$G_{xy}$   
DWI<sub>xy</sub>



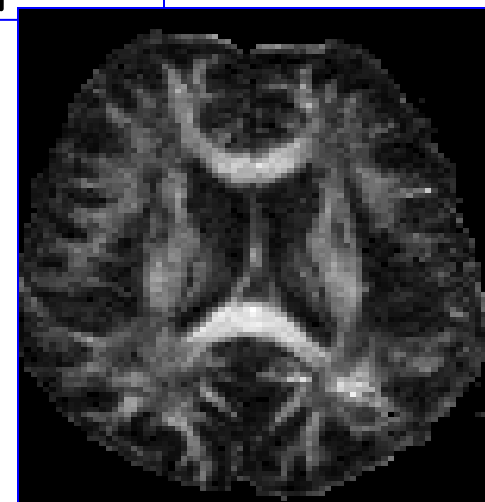
$G_{yz}$   
DWI<sub>yz</sub>

MD and FA maps

$$\bar{D} = MD = \frac{D_1 + D_2 + D_3}{3}$$



MD map



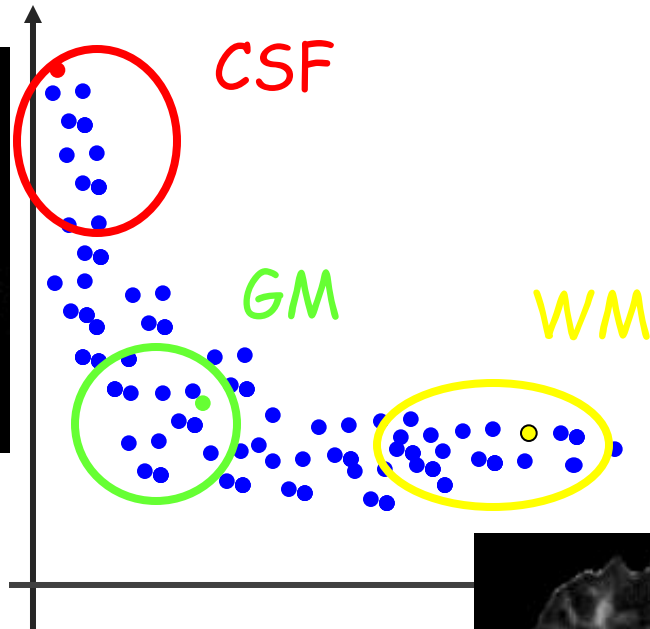
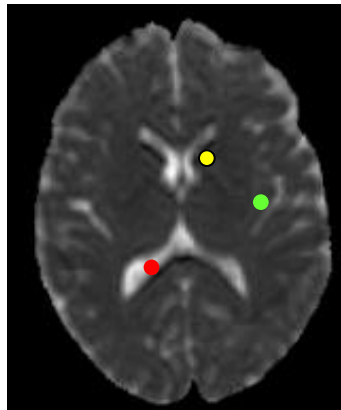
FA map

$$FA = \frac{\sqrt{3[(D_1 - \bar{D})^2 + (D_2 - \bar{D})^2 + (D_3 - \bar{D})^2]}}{\sqrt{2(D_1^2 + D_2^2 + D_3^2)}}$$

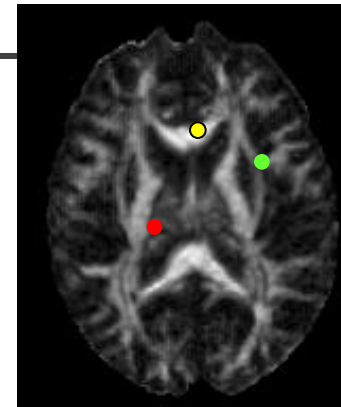
MD and FA  
Quantitative images

### Diffusion Tensor Imaging

mean diffusivity

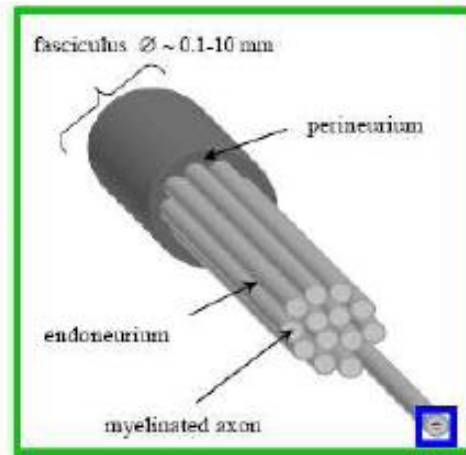
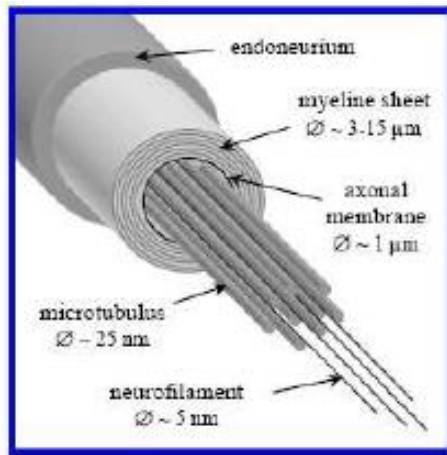
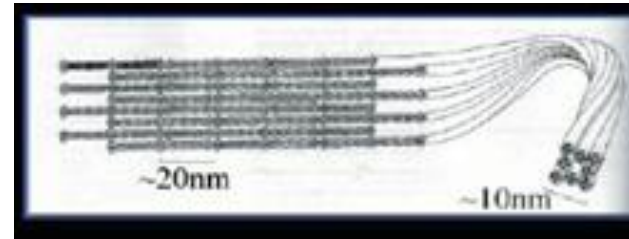
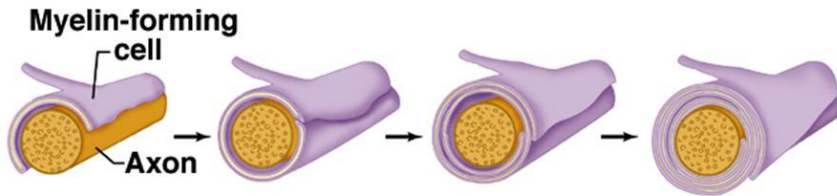


anisotropy



Low sensitivity and specificity

# Axon in successive stages of myelinization



Axon mean diameter  $\approx 7\mu\text{m}$   
 Microtubulus  $\approx 20\text{nm}$

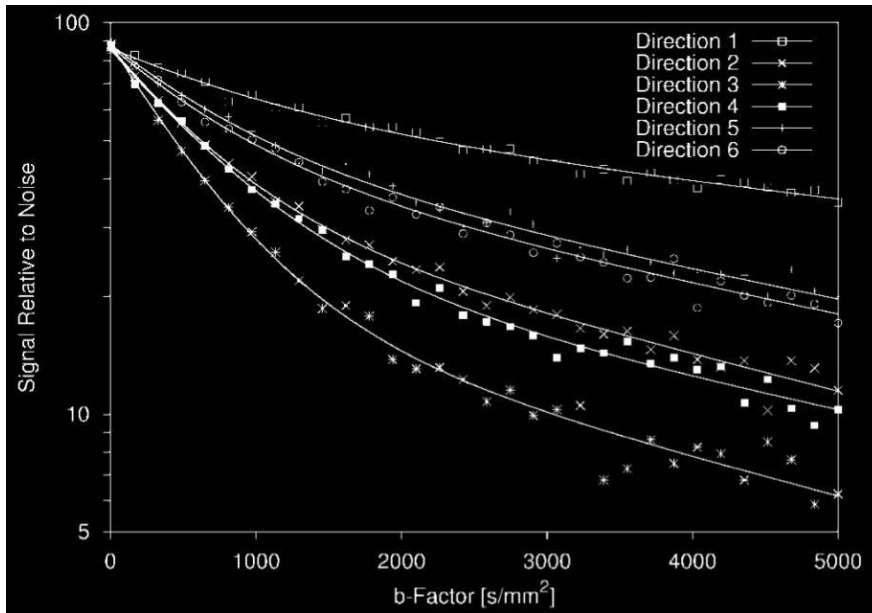
$$\langle r^2 \rangle = 6 D \Delta$$

$$(\langle r^2 \rangle)^{1/2} \approx 30\mu\text{m}$$

Where:  $D \approx 1 \times 10^{-9} \text{ m}^2/\text{s}$   
 $\Delta = 80\text{ms}$

## Prostate Cancer staging





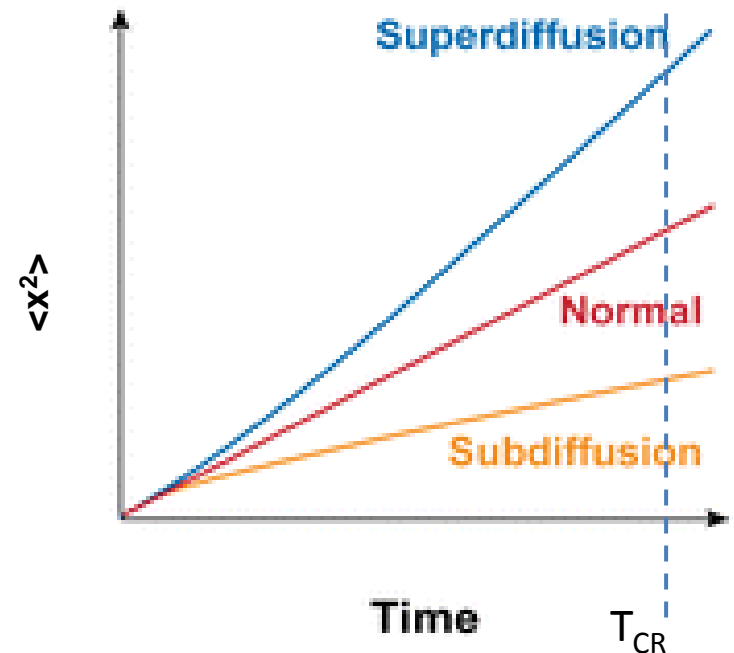
## Anomalous Diffusion

$$\langle r^2(t) \rangle \approx t^\nu$$

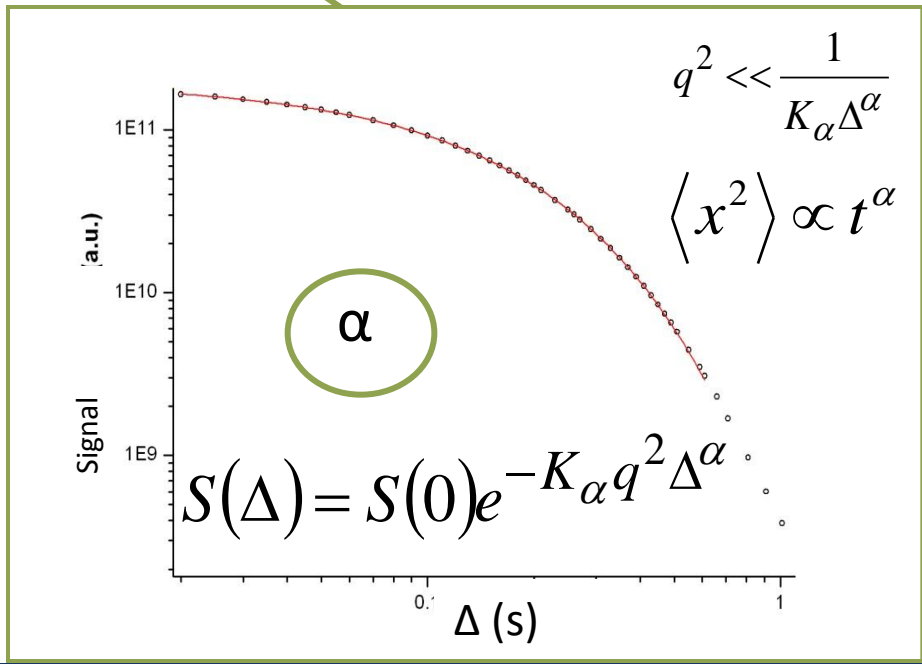
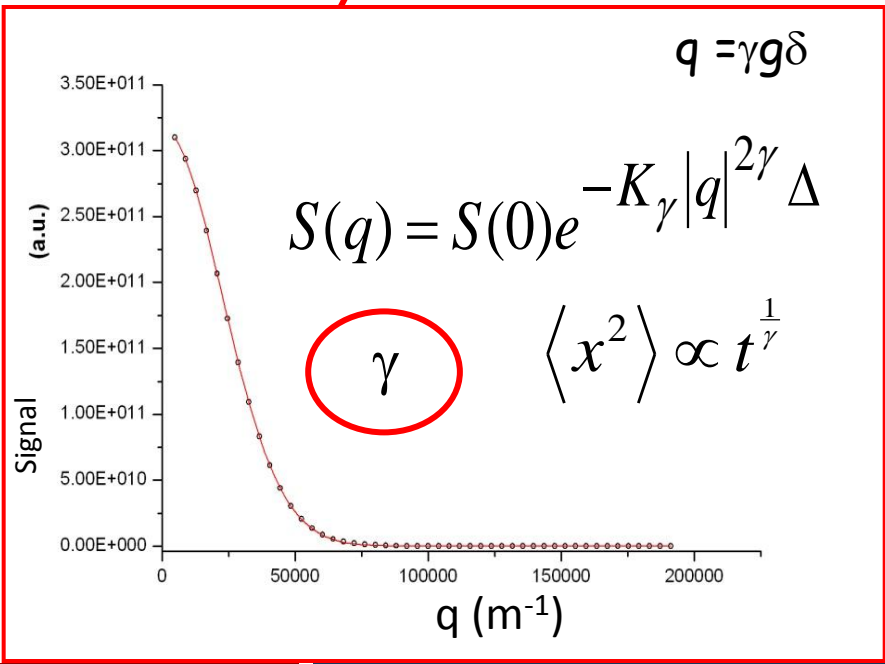
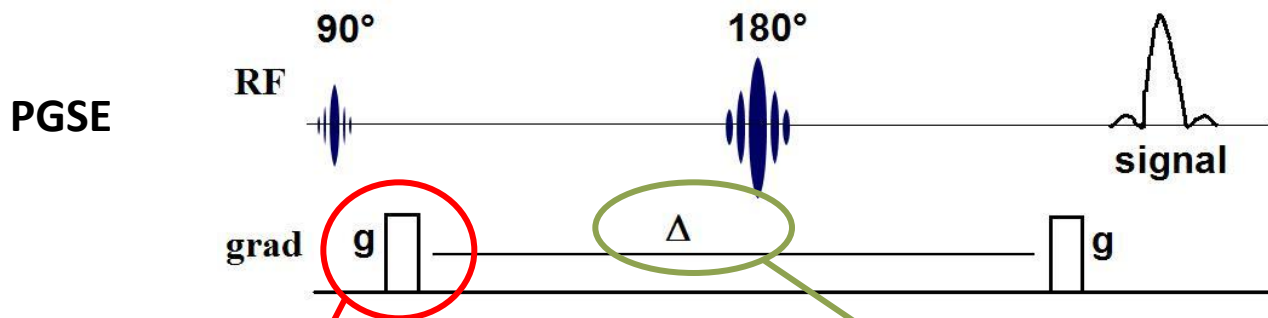
$\nu < 1$  subdiffusion

$\nu > 1$  superdiffusion

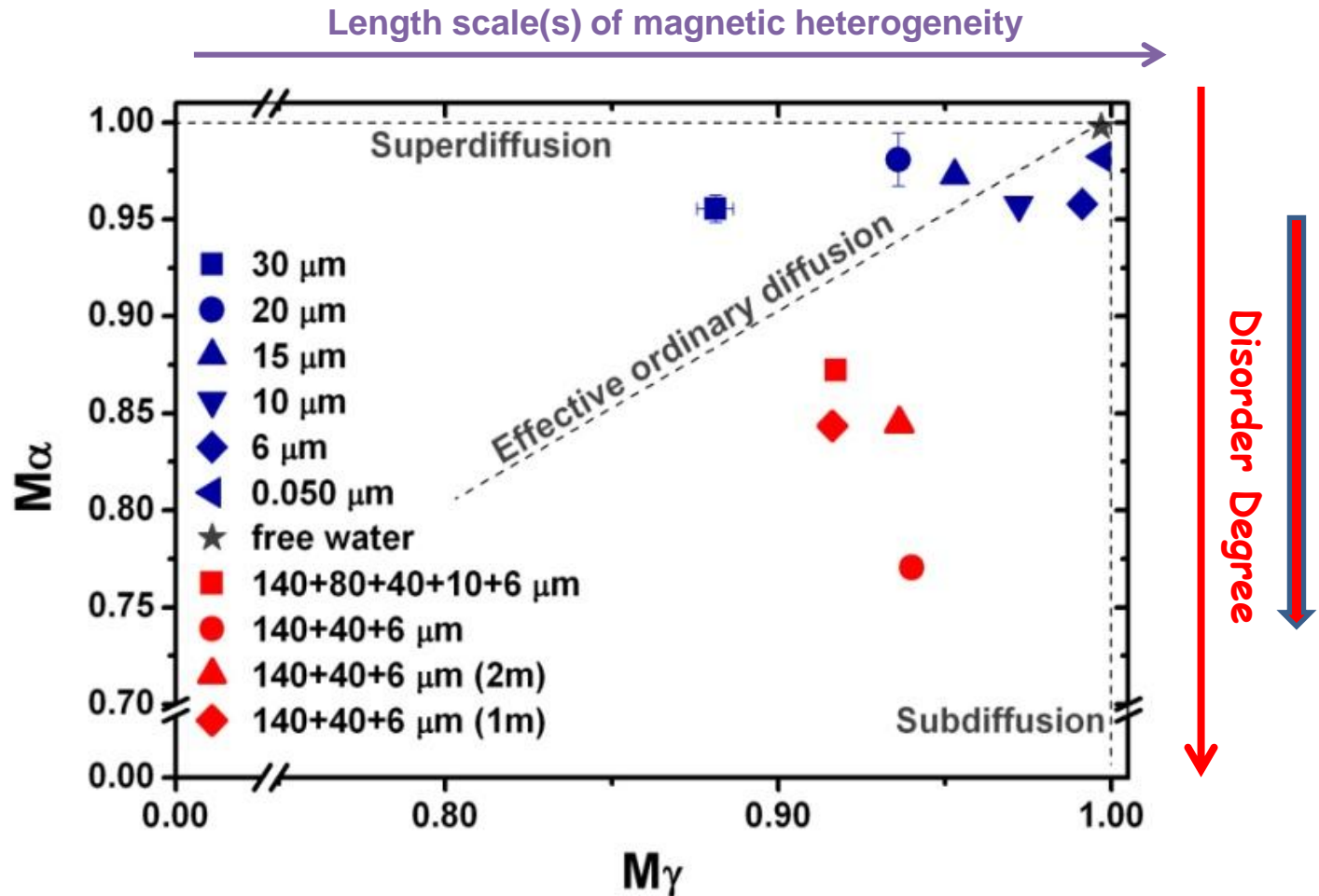
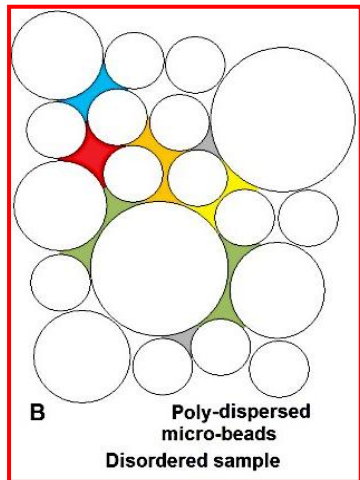
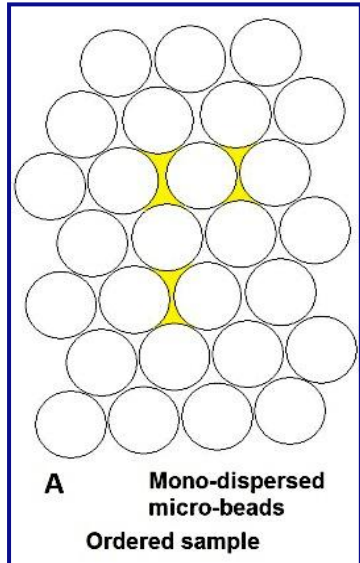
**Transient anomalous diffusion** can be defined when  $MSD = T_D^\alpha$  with  $\alpha < 1$  for  $t \ll T_{CR}$  and  $MSD = T_D$  for  $t \gg T_{CR}$  where  $T_{CR}$  the crossover time.



How to determine  $\alpha$  and  $\gamma$



## (Bio)-physical interpretation of $\alpha$ and $\gamma$ parameters





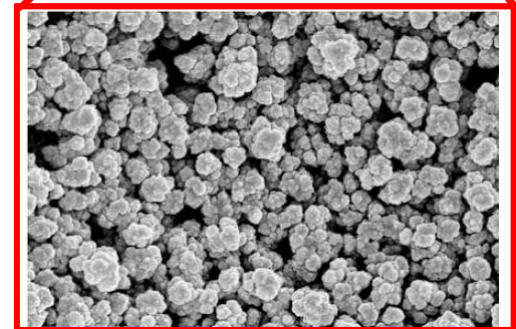
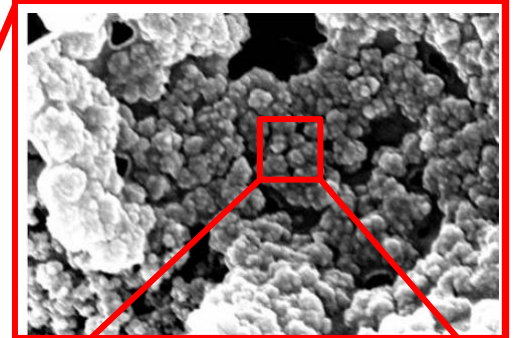
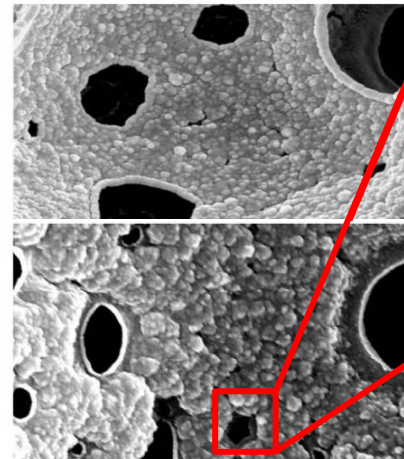
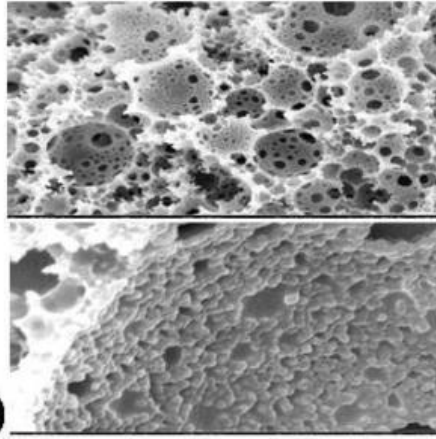
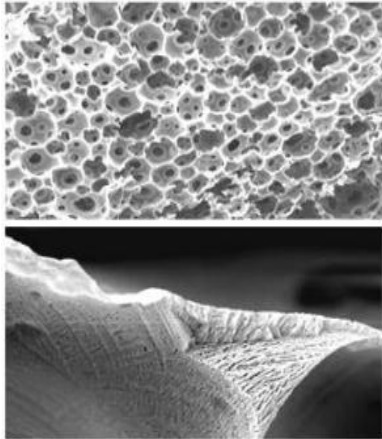
Highly porous polymeric matrices with randomly oriented interconnected pores obtained from a solution of polyvinyl alcohol and etyltrimethylammonium bromide (PVA scaffolds)

- void size distribution: 10-100  $\mu\text{m}$
- interconnection size distribution: 4-50  $\mu\text{m}$
- the three scaffolds differ in the roughness of the walls of their voids and interconnections

PVA1

PVA2

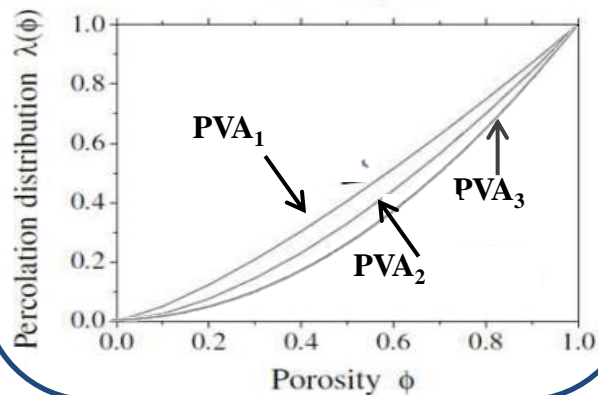
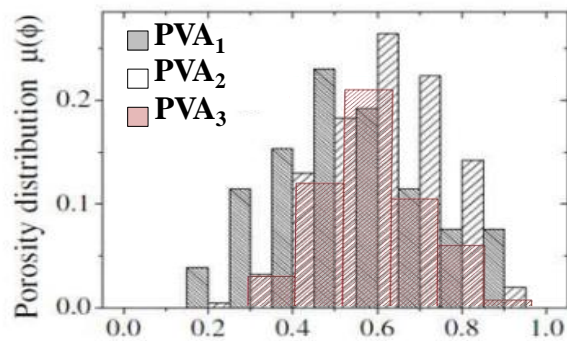
PVA3



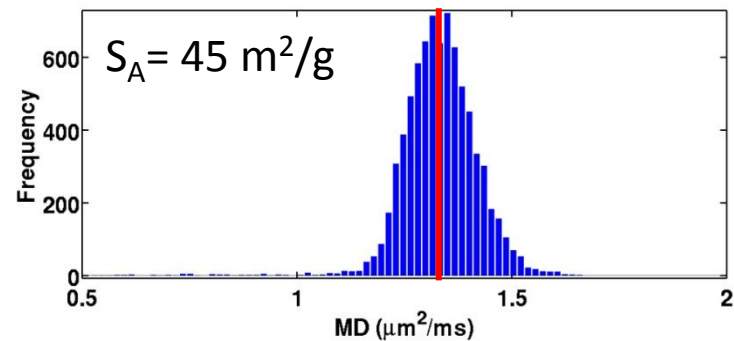
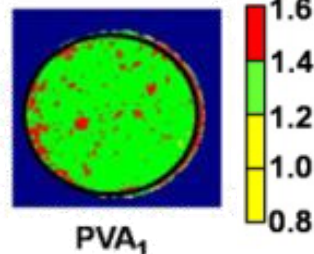
Sample	Surface area ( $\text{m}^2/\text{g}$ ) <sup>a</sup>	$V_p$ ( $\text{cm}^3/\text{g}$ ) <sup>b</sup>	$\langle D_p \rangle$ ( $\text{\AA}$ ) <sup>b</sup>
PVA3	106	0.36	127
PVA2	68	0.38	215
PVA1	45	0.20	104

## Imaging:

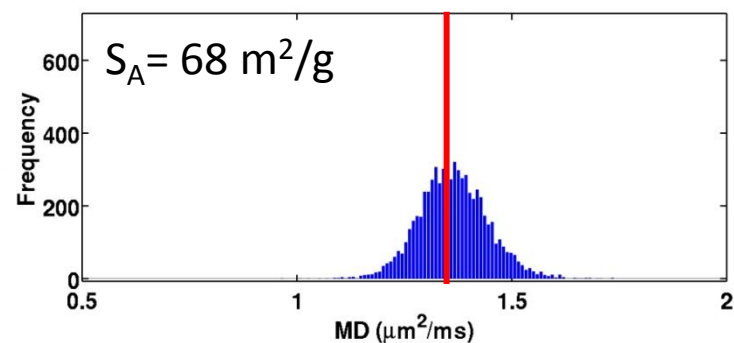
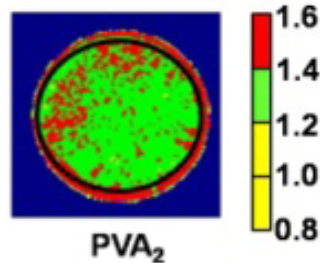
PGSTE sequence  
 $TR = (5000 - \Delta)$  ms  
 $TE = 15$  ms;  
 $\delta = 2$  ms  
 $g = 74$  mT/m  
 $\Delta = (20 \div 520)$  ms



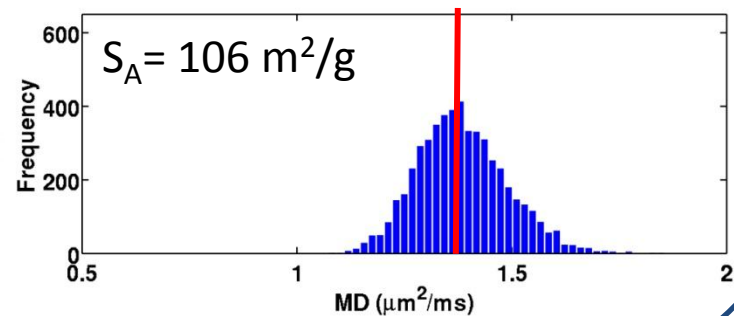
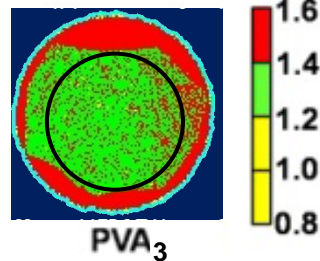
MD-map  
( $\mu\text{m}^2/\text{ms}$ )



MD-map  
( $\mu\text{m}^2/\text{ms}$ )



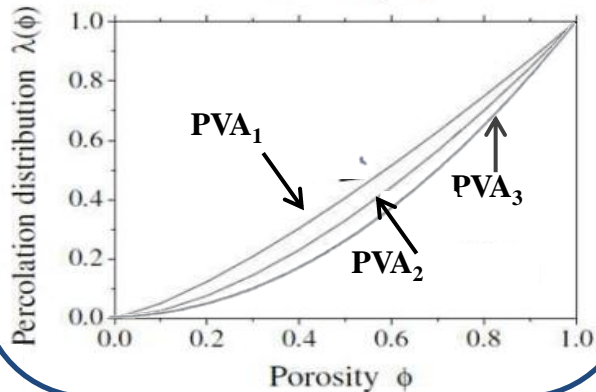
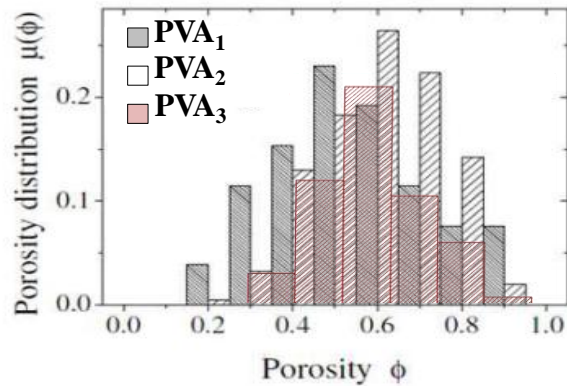
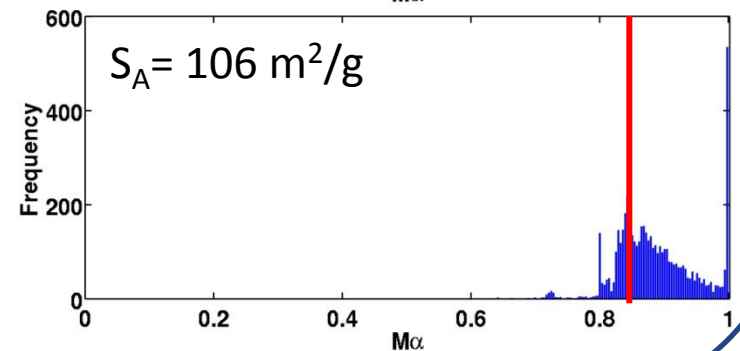
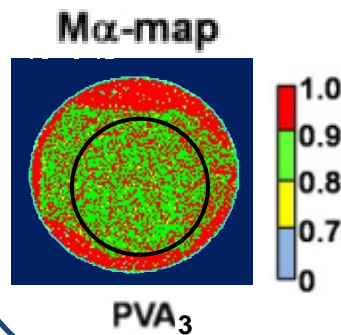
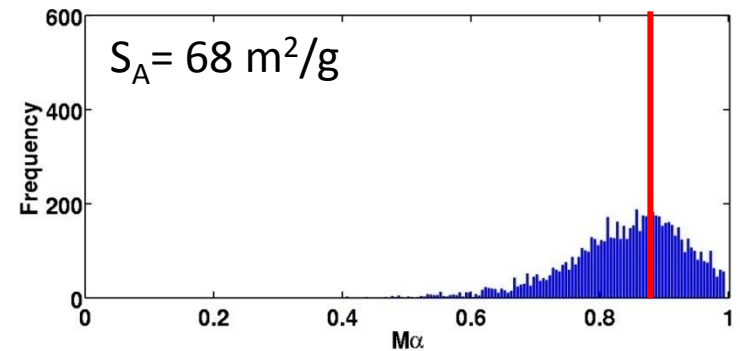
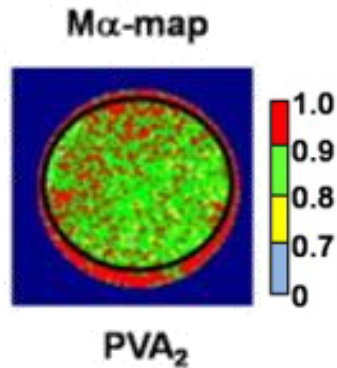
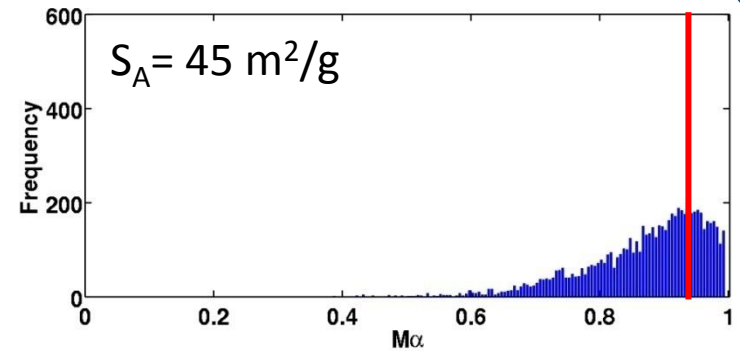
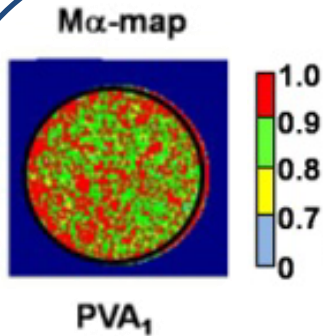
MD-map  
( $\mu\text{m}^2/\text{ms}$ )



# Anomalous diffusion approach

## Imaging:

PGSTE sequence  
 $TR = (5000 - \Delta)$  ms  
 $TE = 15$  ms;  
 $\delta = 2$  ms  
 $g = 74$  mT/m  
 $\Delta = (20 \div 520)$  ms

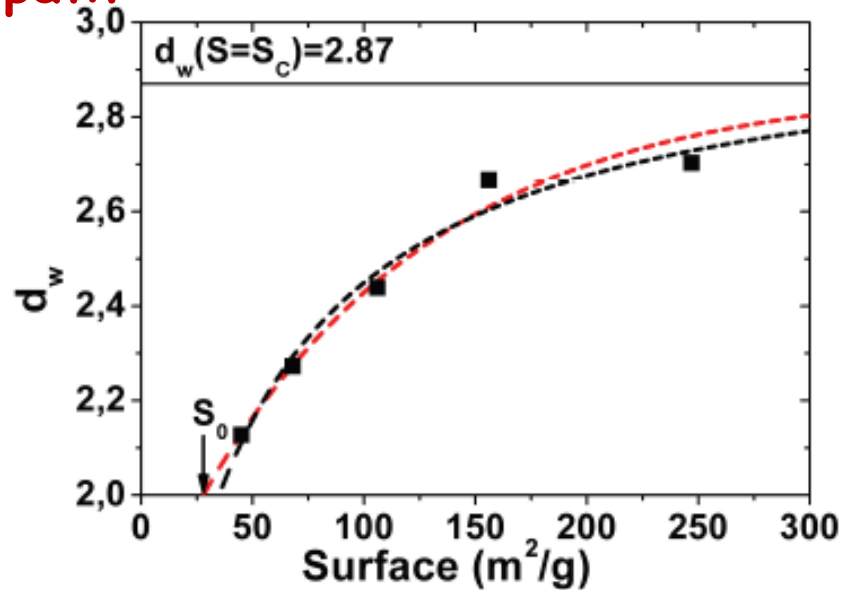


# Application to multiscale porous materials

fractal dimension of the random path

$$d_w = 2/a$$

$\alpha$  quantifies global structural complexity



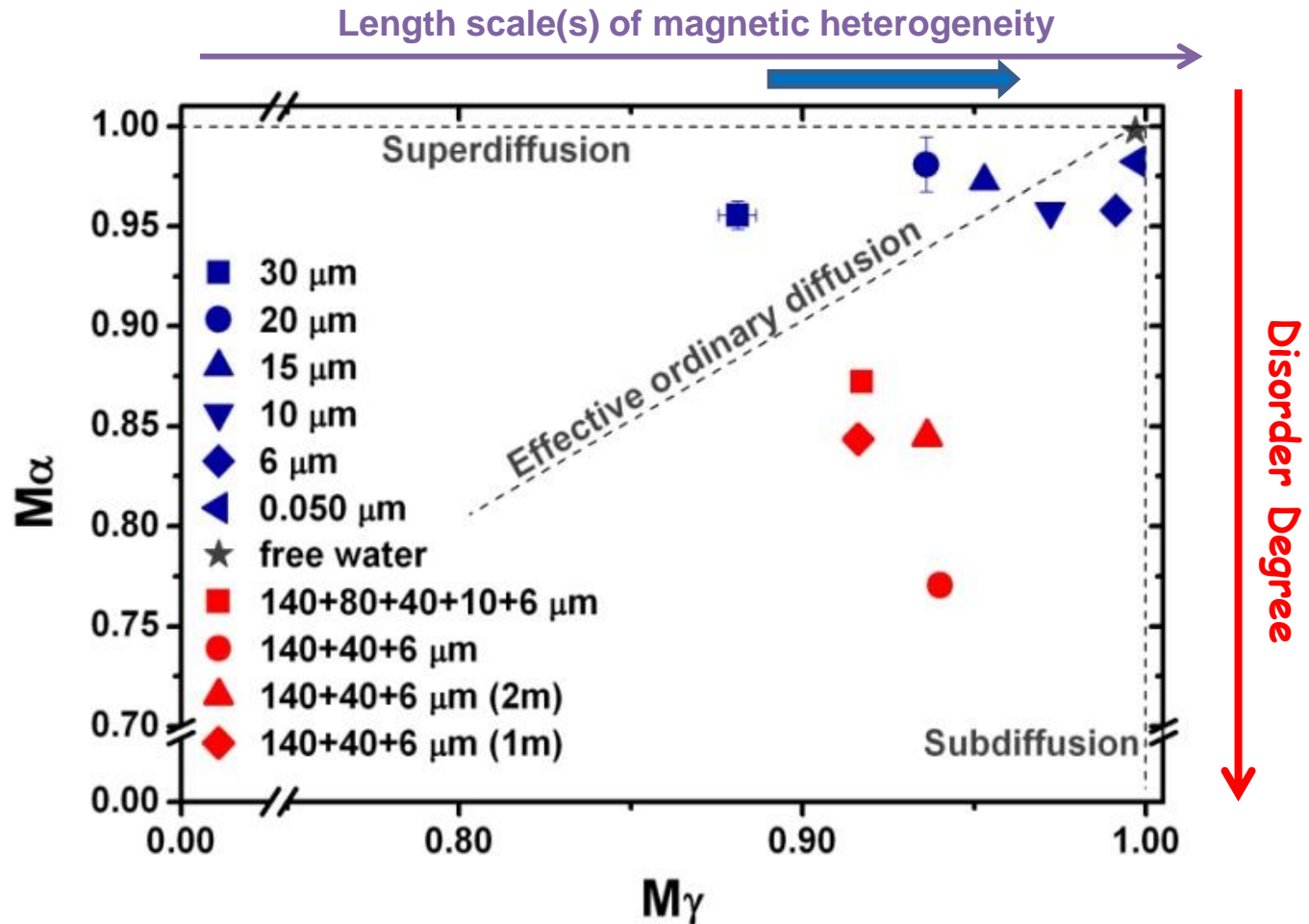
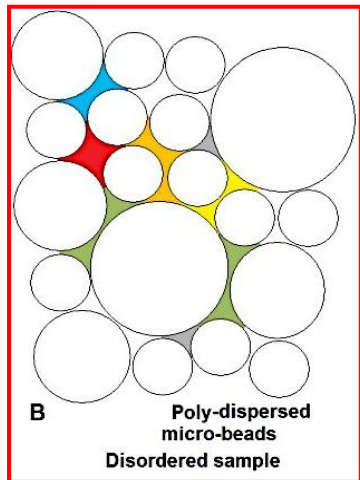
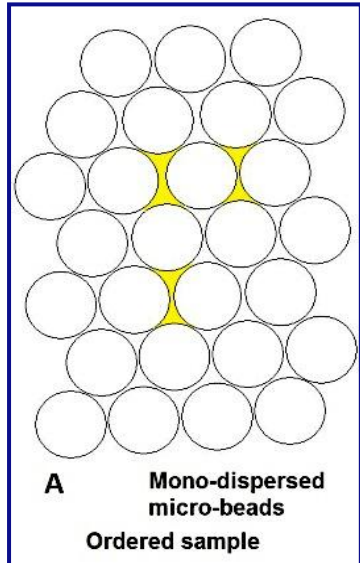
$$d_w = \frac{A1+B1x+C1x^2}{A2+B2x+C2x^2}$$

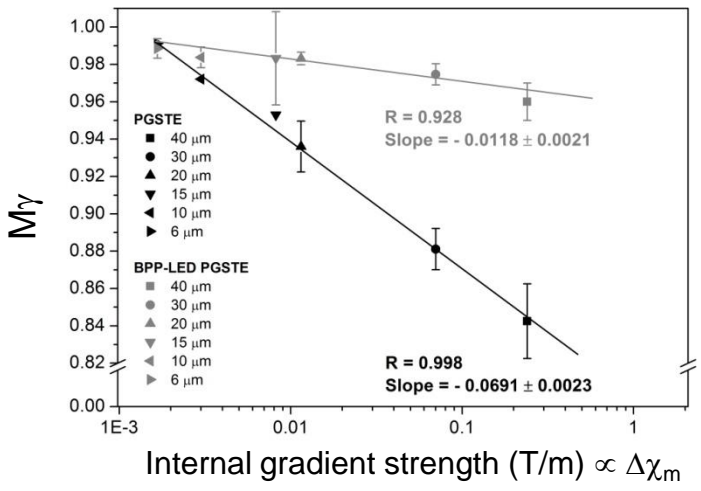
M.J. Saxton

*Biophys, J.* 66 (1994)

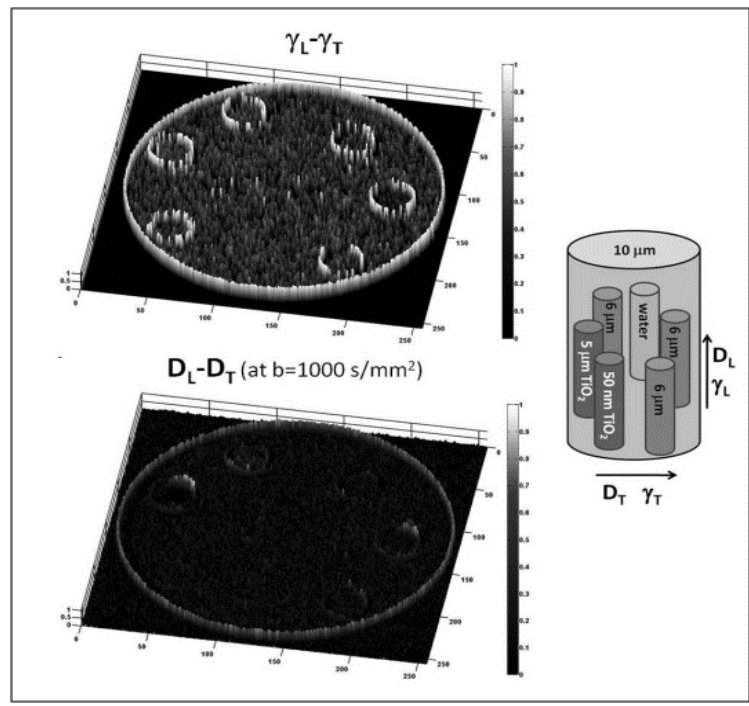
$x=S/\sigma$

## (Bio)-physical interpretation of $\alpha$ and $\gamma$ parameters





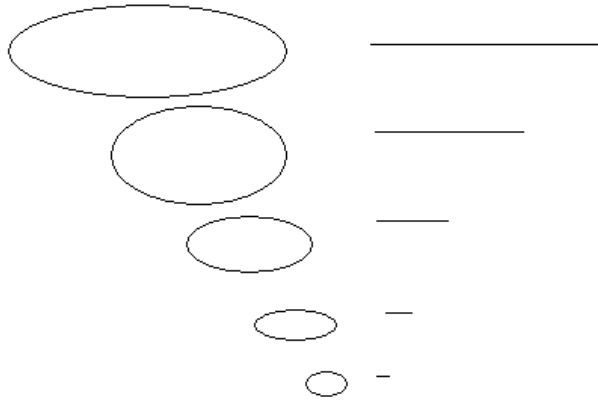
$$B_{ind} \sim \Delta\chi * B * A * \cos\theta$$



Increase of the image contrast  
Enhanced interface contrast

# Pseudo-superdiffusion

## multi-compartmentalization



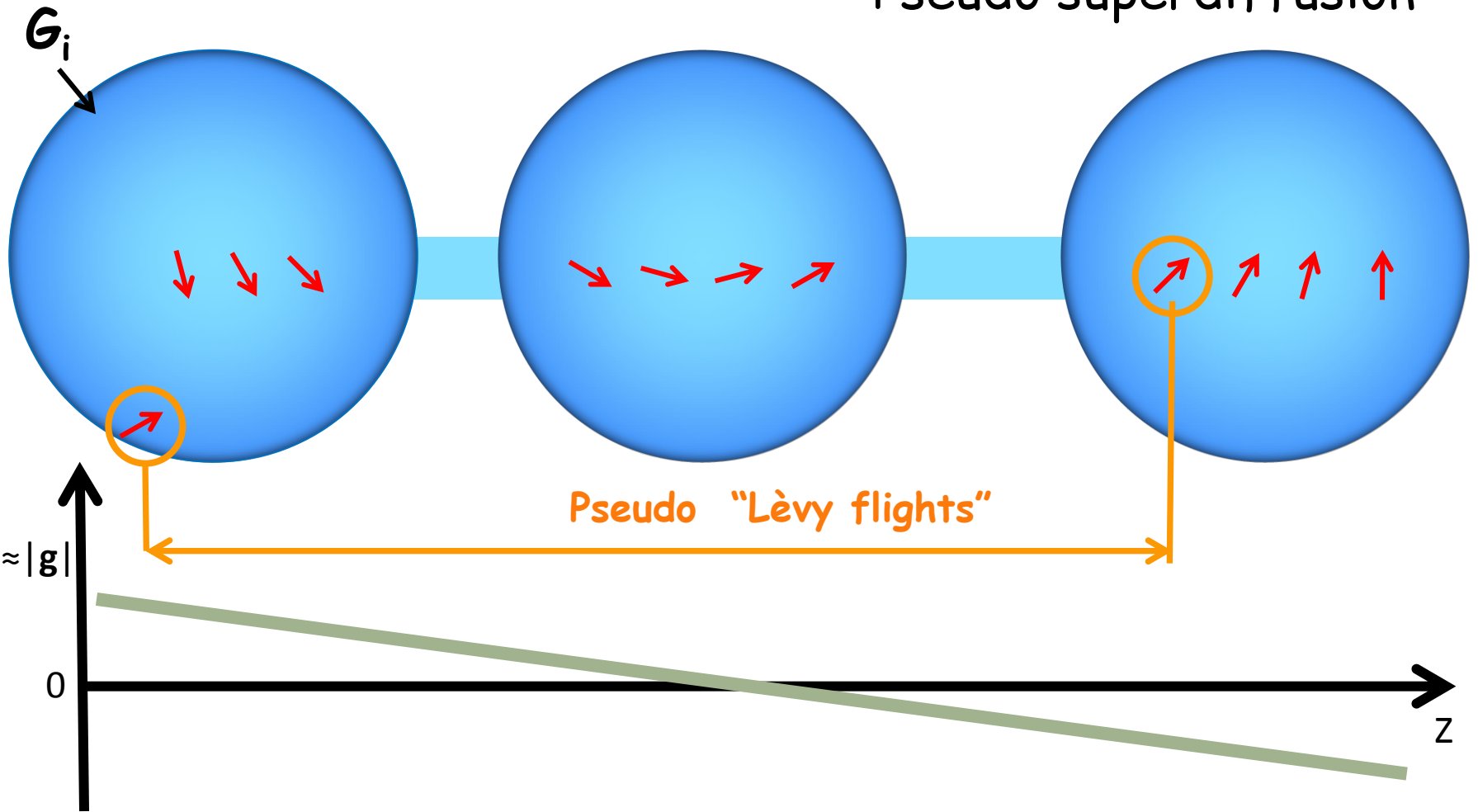
intravoxel diffusion heterogeneity in space, i.e., water molecules diffuse with considerably different free lengths.

due to both water **multi-compartmentalization** and **magnetic susceptibility differences ( $\Delta\chi$ )** at the interface between different compartments

## Multi-compartmentalization + magnetic susceptibility differences ( $\Delta\chi$ )



# Pseudo superdiffusion

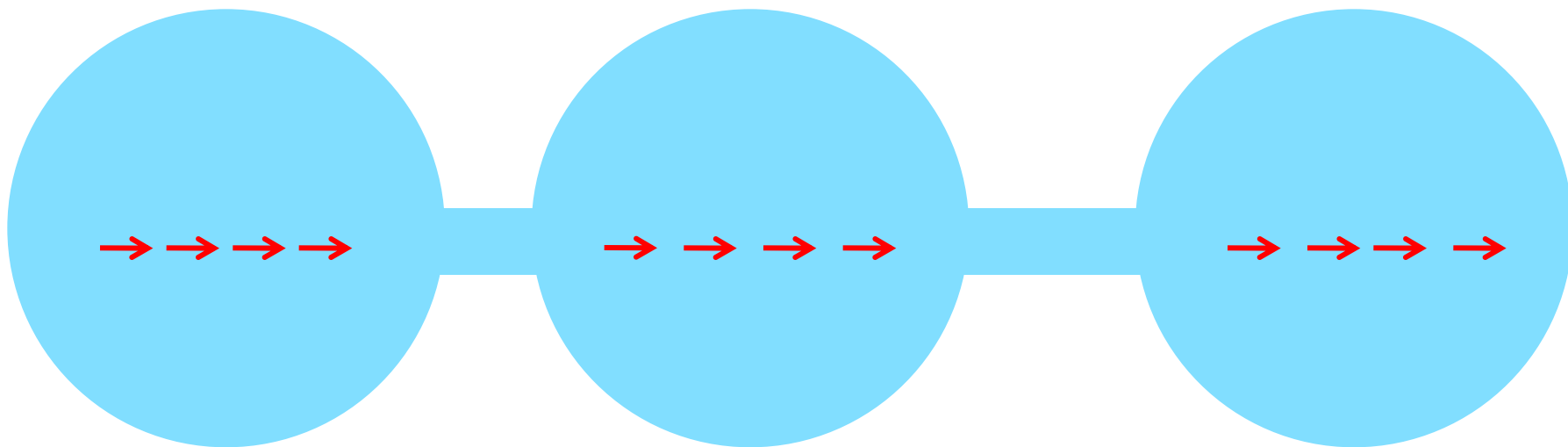


Superdiffusion process ( $\gamma < 1$ ) is spurious, not real but due to local  $\Delta X$  (or  $G_i$ ) at the interface between beads and water

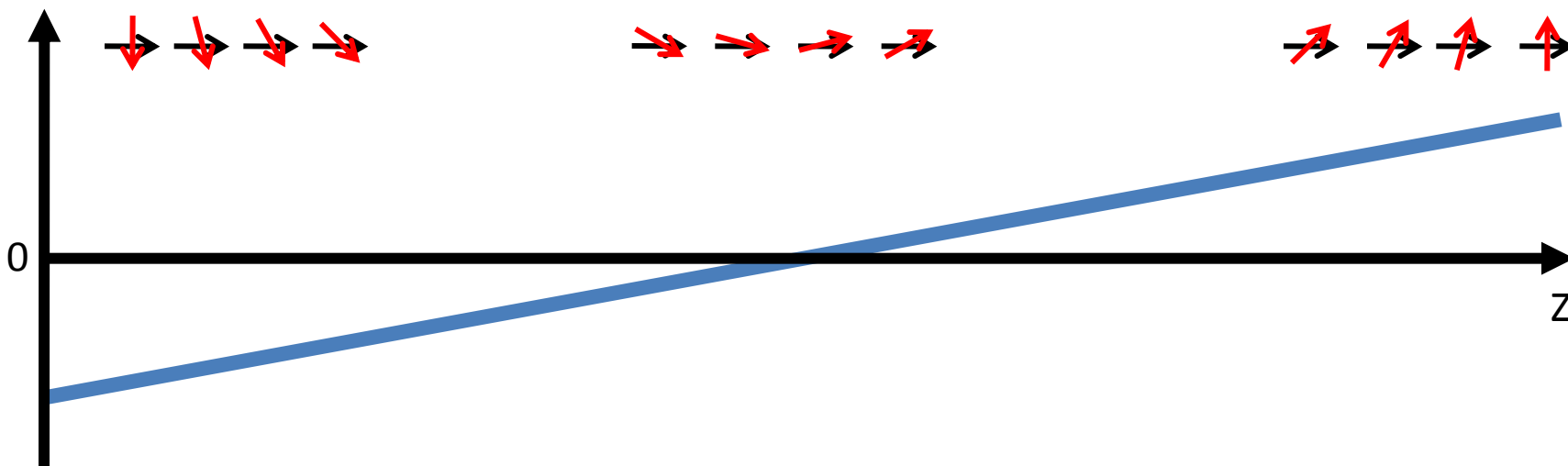


Without boundary effects (no  $G_i$ )

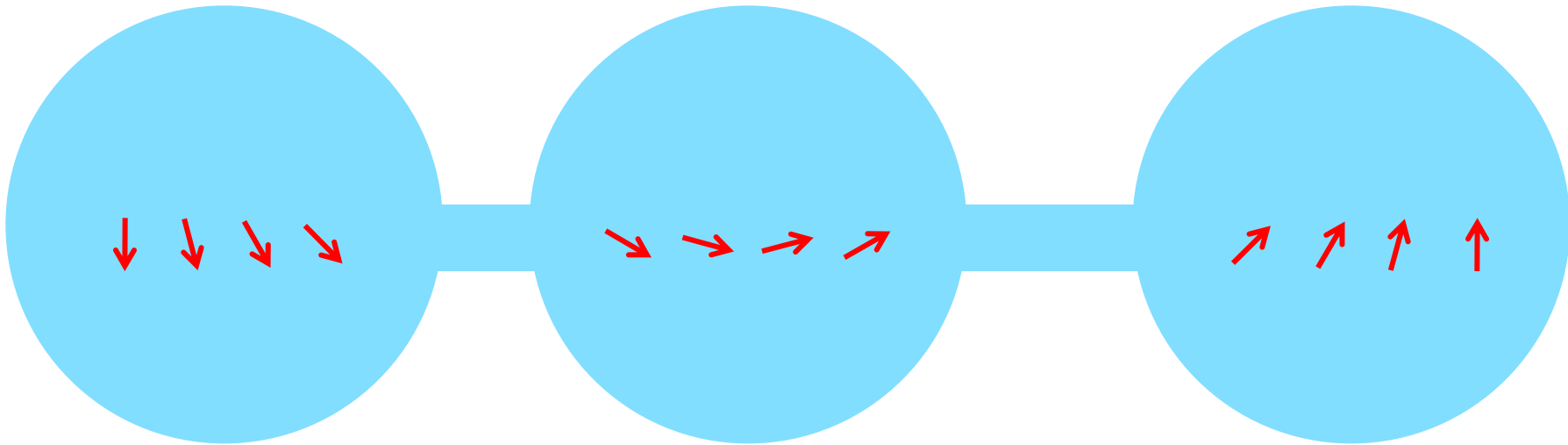
**Brownian Diffusion**



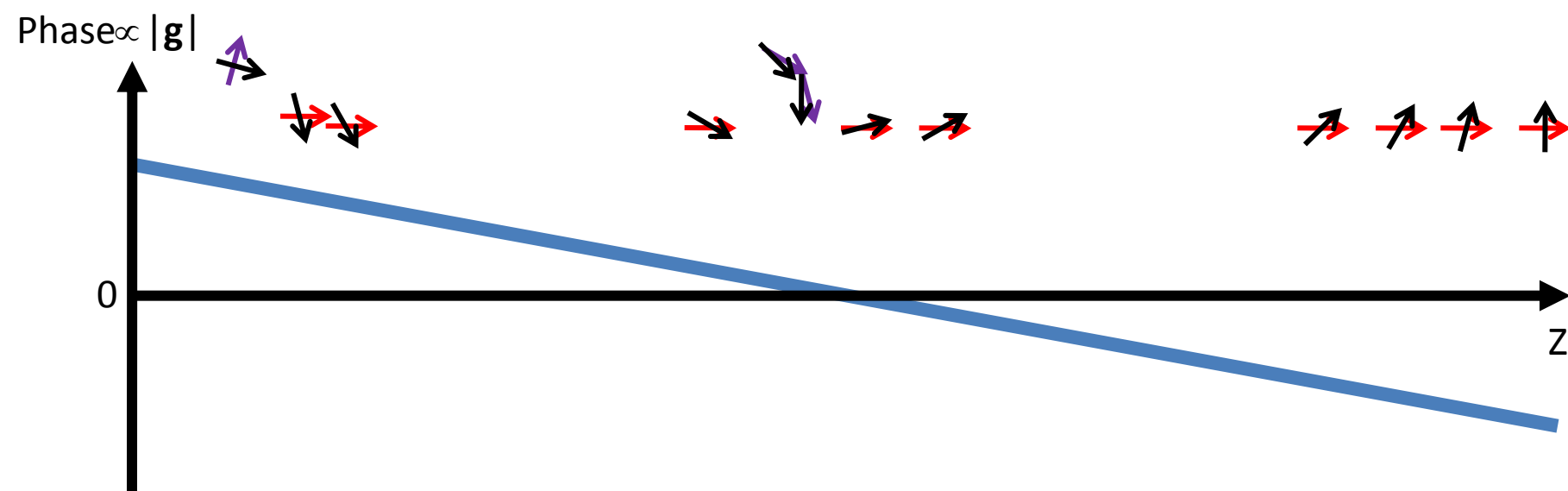
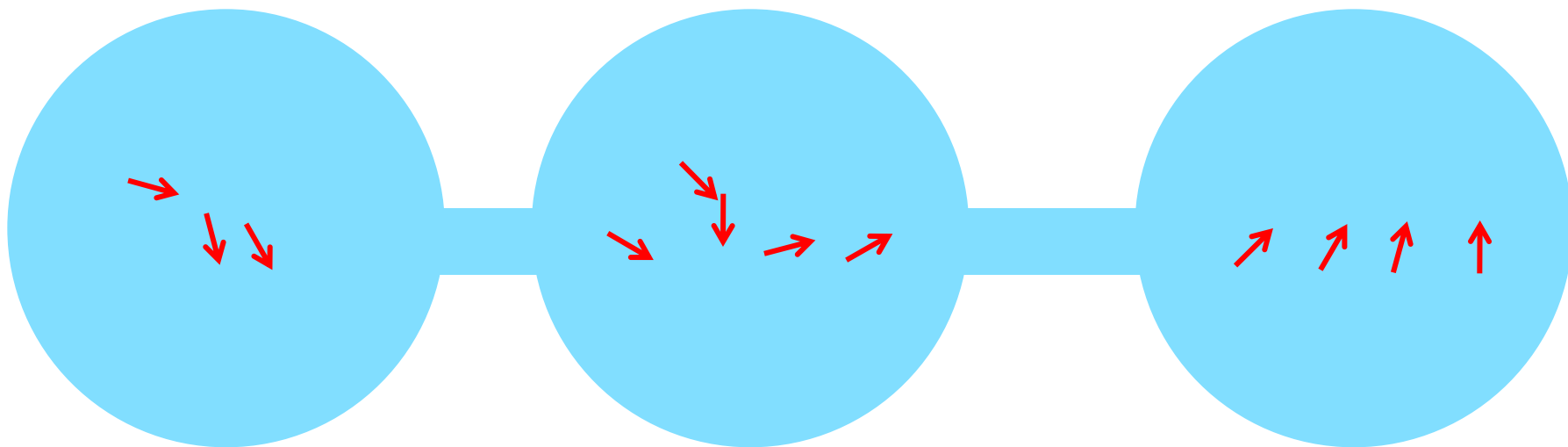
Phase  $\Delta\phi \propto |\mathbf{g}|$



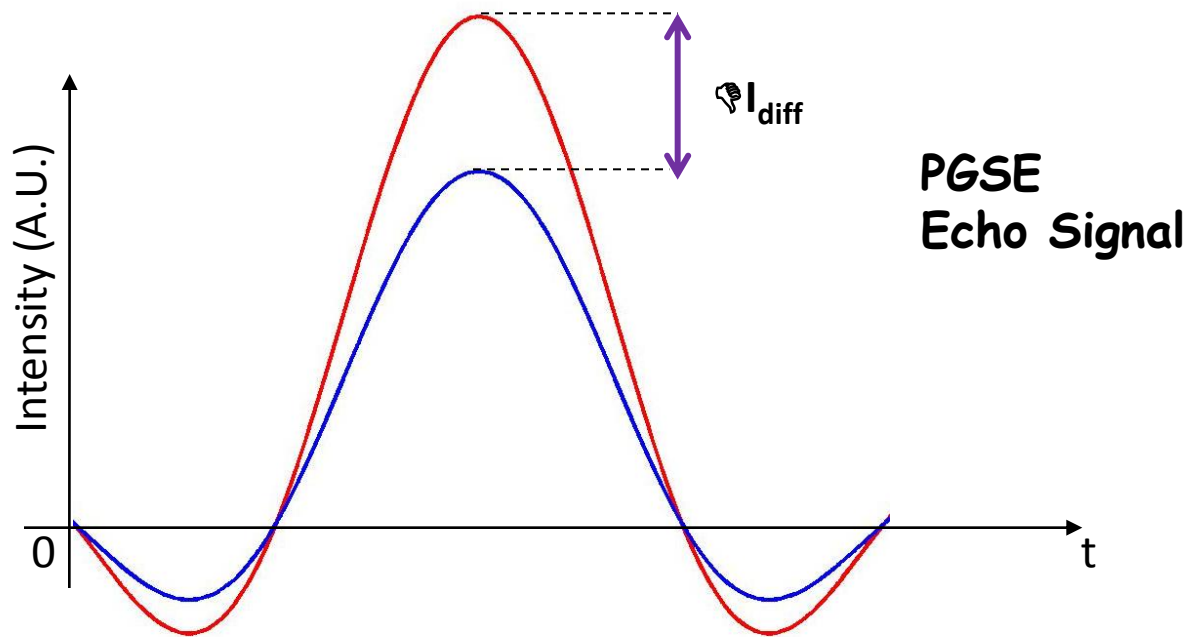
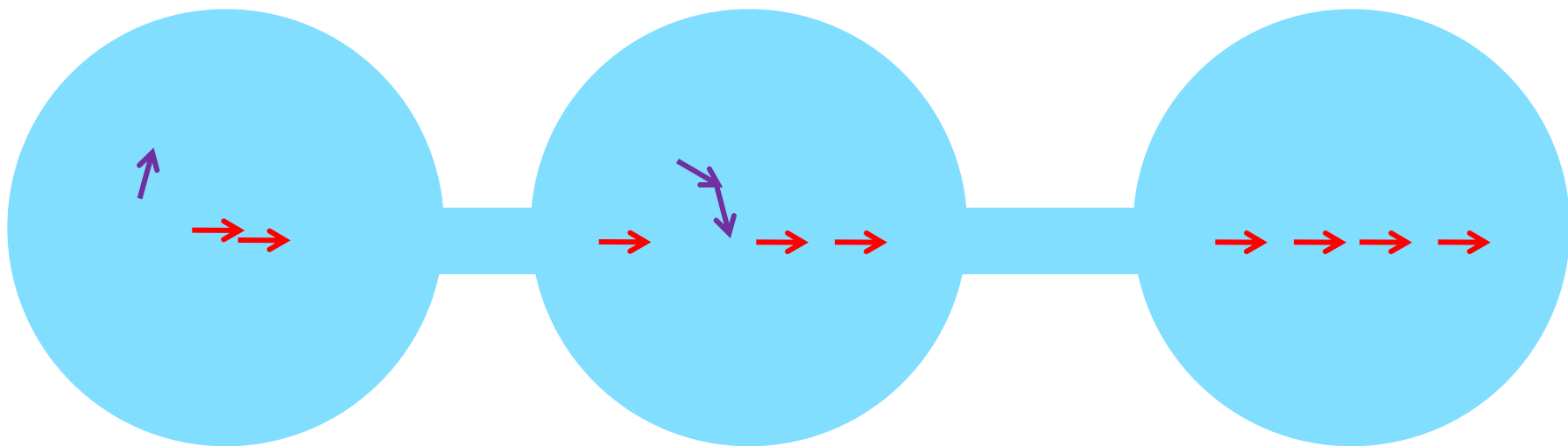
# Brownian Diffusion



# Brownian Diffusion

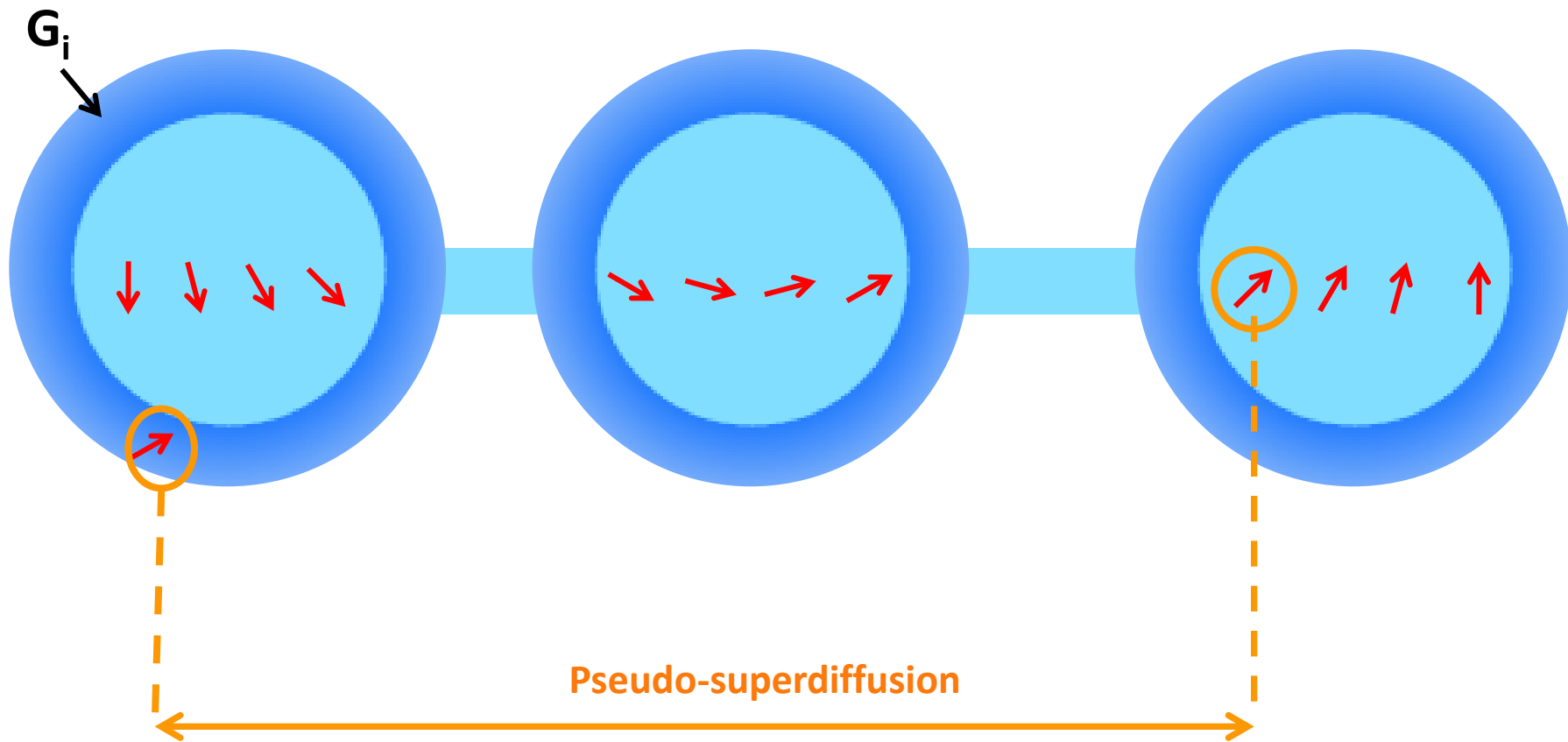


# Spectroscopy, Brownian Diffusion

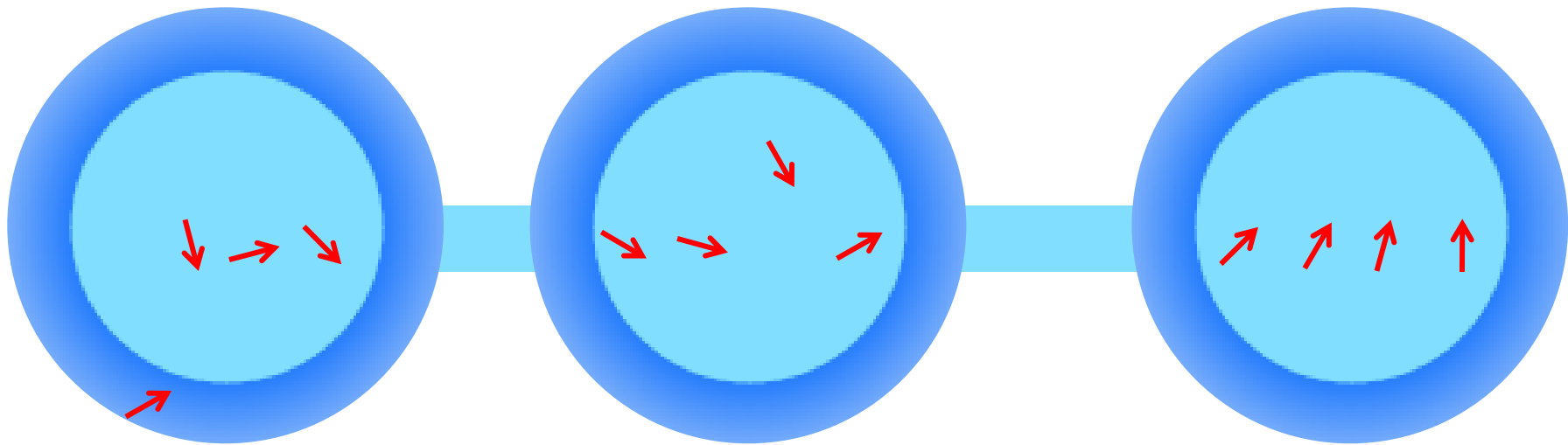


With boundary effects:  $G_i$

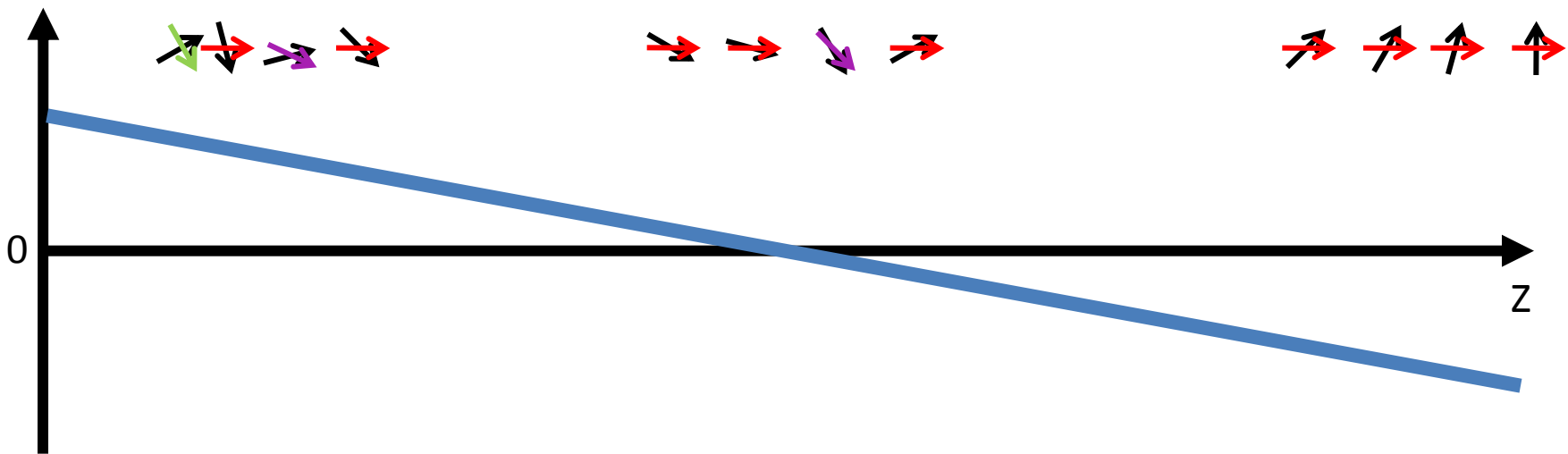
Spectroscopy pseudo-superdiffusion



# Spectroscopy pseudo-superdiffusion

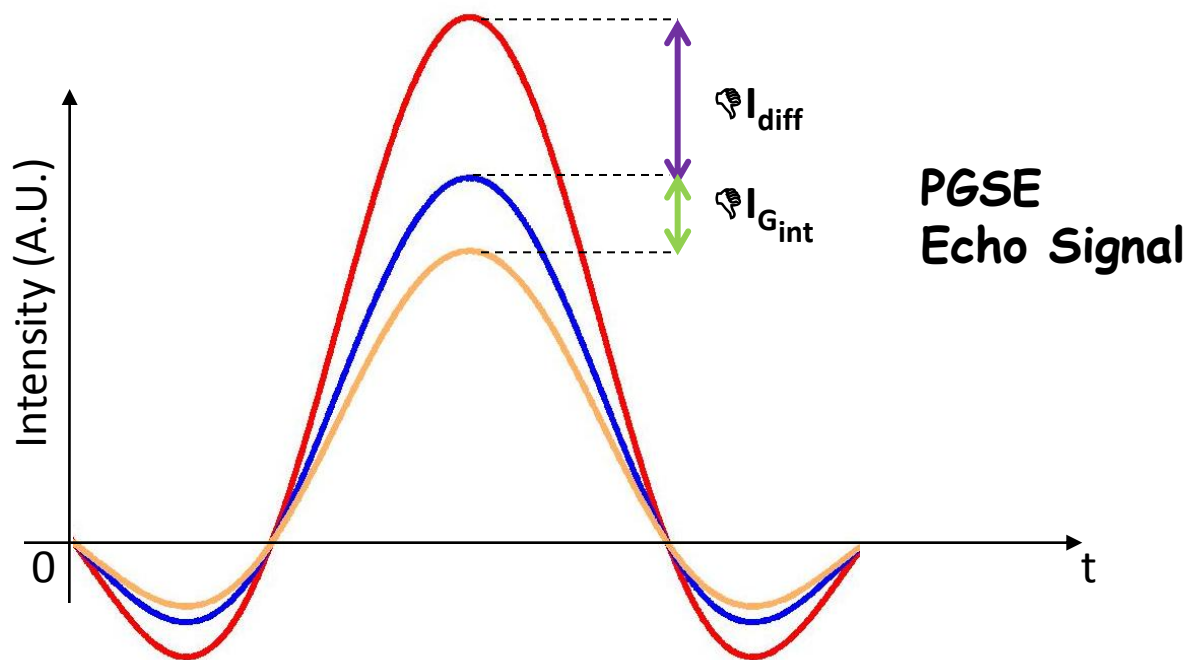
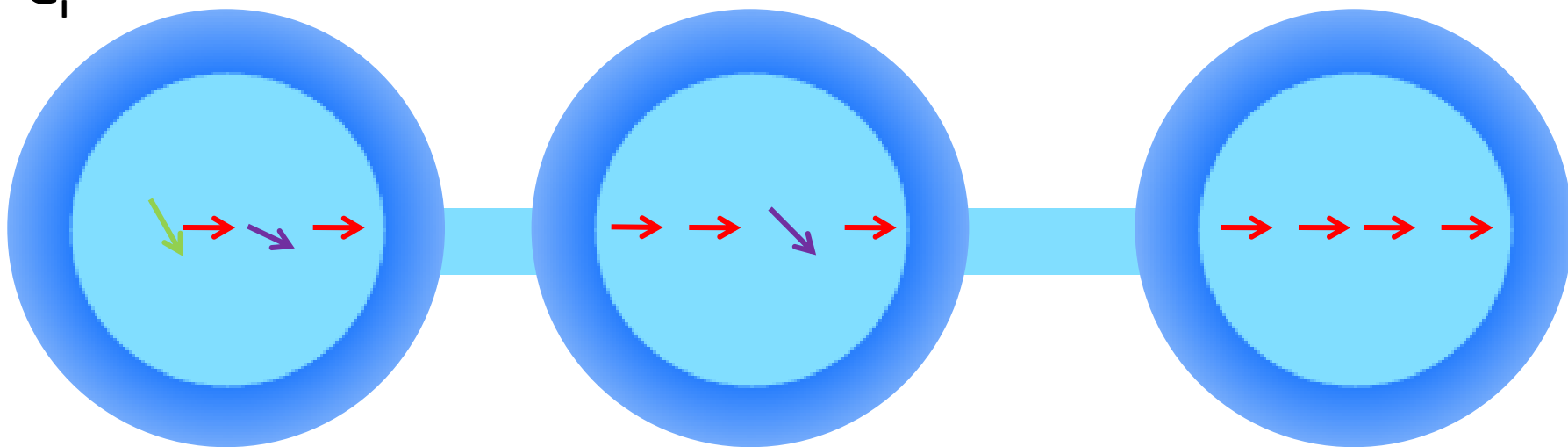


Phase  $\phi|g|$



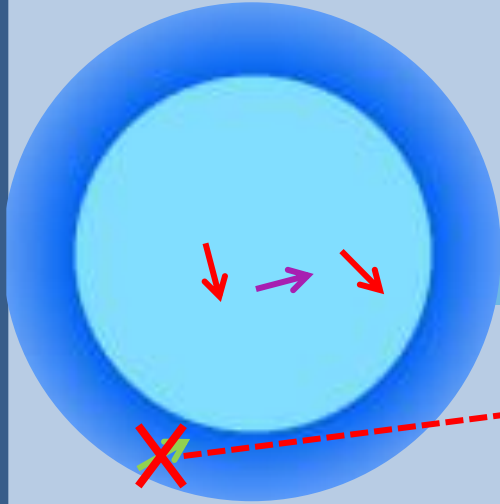
# Pseudo-Super Diffusion

$G_i$



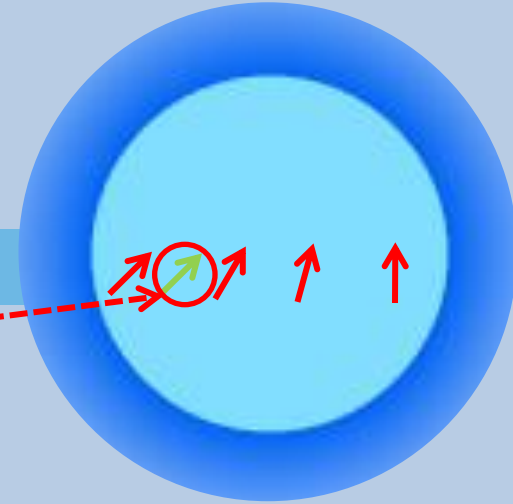
# Pseudo-Super Diffusion Imaging

Voxel 1



$G_i$

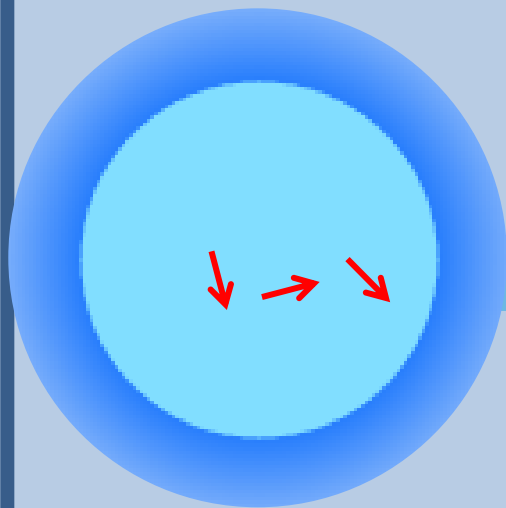
Voxel 2



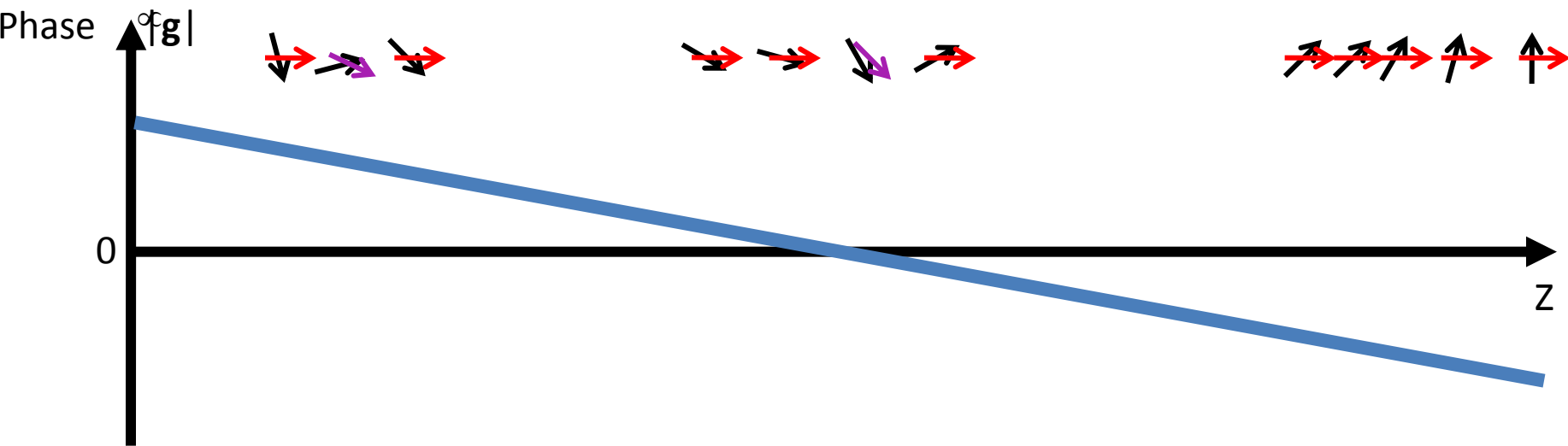
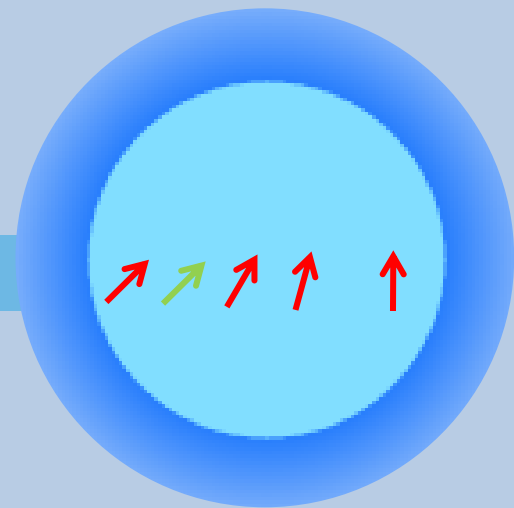
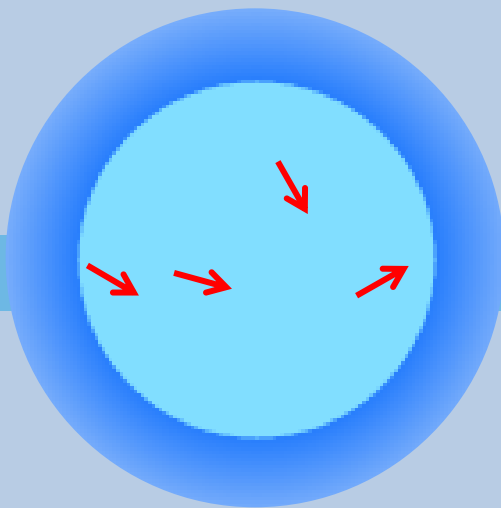


# Pseudo-Super Diffusion Imaging

Voxel 1

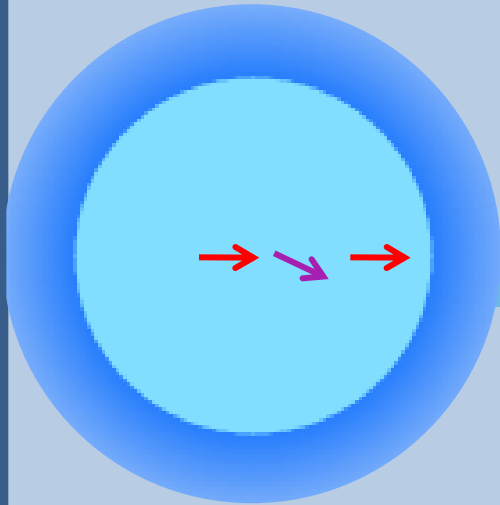


Voxel 2

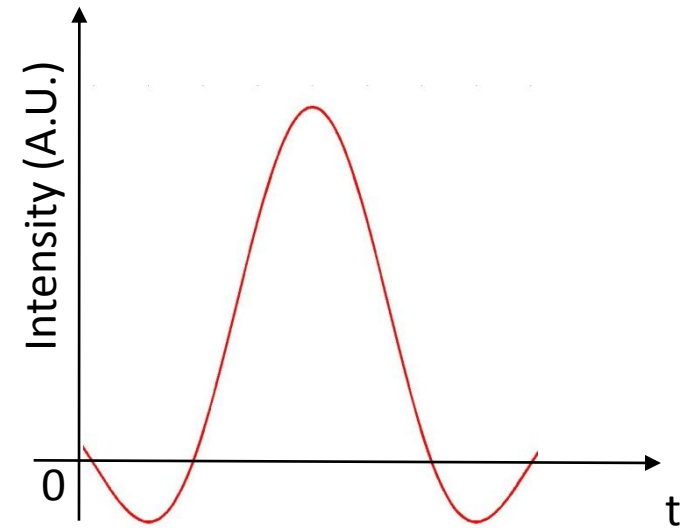
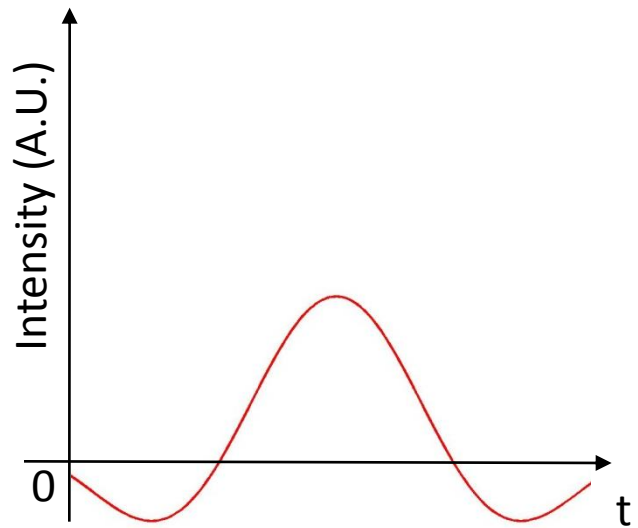
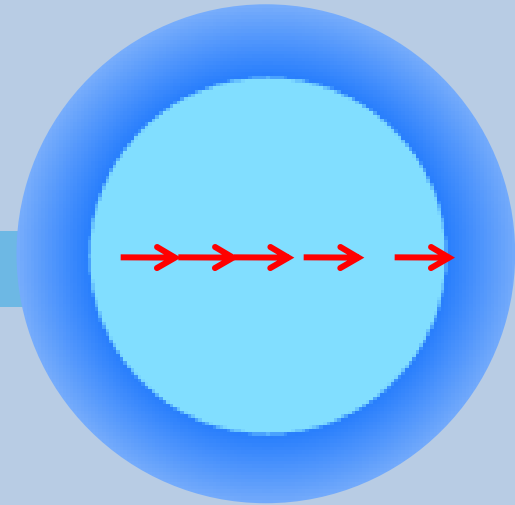
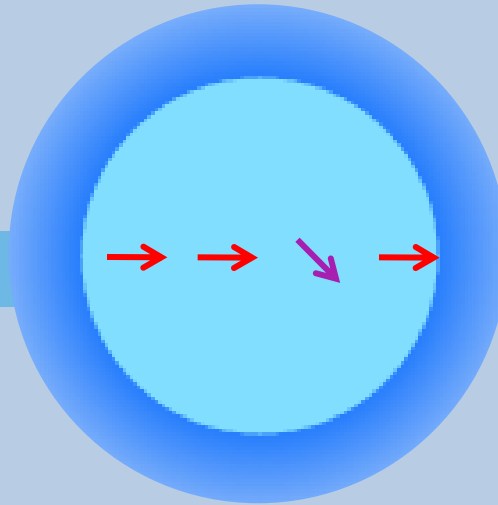


# Pseudo-Super Diffusion Imaging

Voxel 1

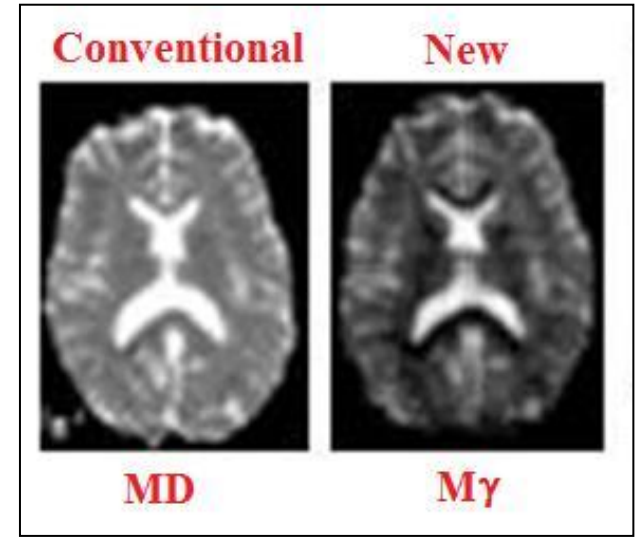
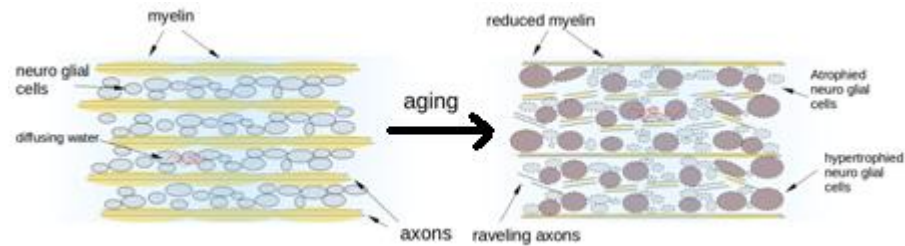
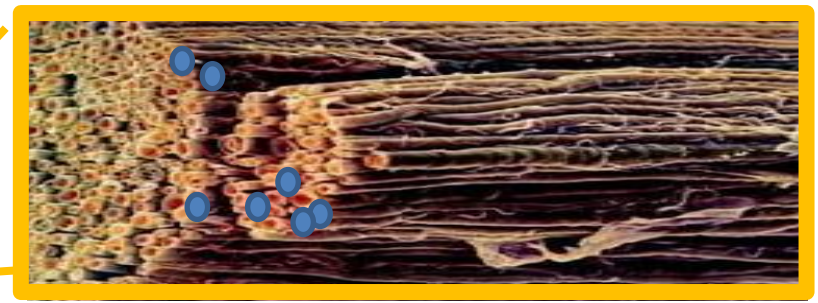
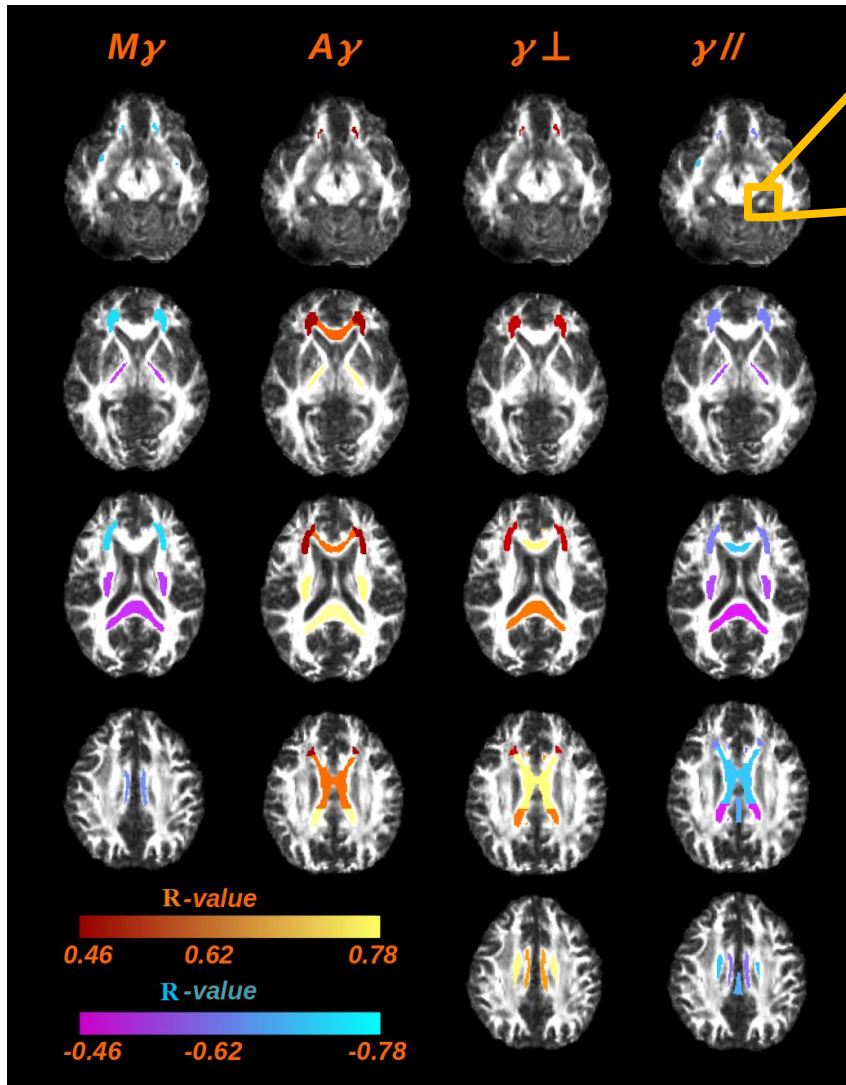


Voxel 2

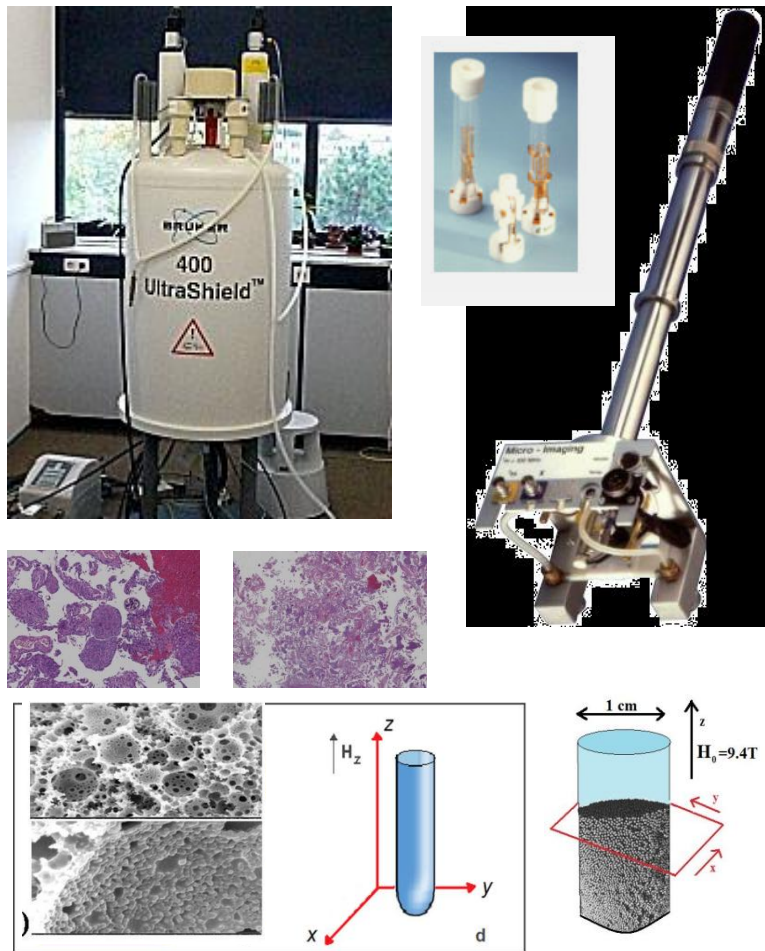


Diffusion reflect the micro-structural rearrangement of tissues

## BRAIN AGING



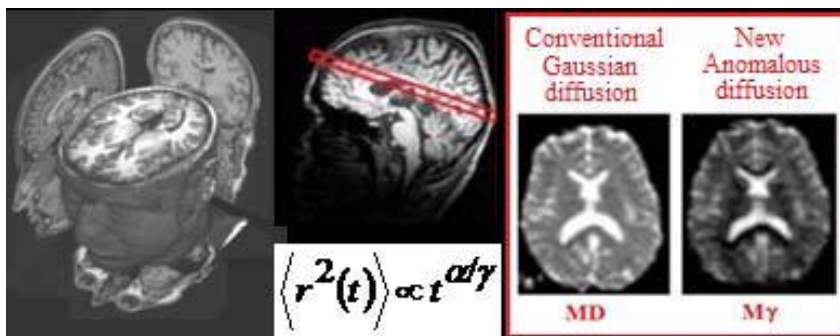
To investigate samples...



To investigate Human subjects...



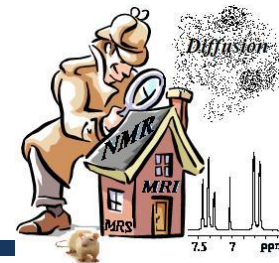
Thank  
you  
for your  
attention



[silvia.capuani@roma1.infn.it](mailto:silvia.capuani@roma1.infn.it)



# NMR Laboratory



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