Eigenvector-Based Centralities for Multiplex and Temporal Networks

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I will discuss a framework *supracentrality* [1, 2, 3] that extends eigenvector-based centralities (including PageRank, eigenvector centrality, and hub/authority scores) to multiplex and temporal networks. Given a network composed of layers $t = 1, 2, 3, \ldots$, our approach involves building centrality matrices $\{\mathbf{C}^{(t)}\}$ and coupling them into a "supracentrality matrix" $\mathbb{C}(\omega) = \text{diag}\left(\mathbf{C}^{(t)}\right) + \omega\tilde{\mathbf{A}} \otimes \mathbf{I}$. Here, $\otimes$ is the Kronecker product, $\omega \geq 0$ denotes the weight of interlayer coupling, and $\tilde{\mathbf{A}}$ denotes an interlayer adjacency matrix that gives the topology of interlayer coupling. Entries in the dominant eigenvector of $\mathbb{C}(\omega)$ give a centrality measure for each node i in each layer t, referred to as a *joint centrality*. We also define *marginal centrality* and *conditional centrality* to allow for a rich characterization of centrality. My talk will focus on new extensions to [1] that explore how *supracentralities* are influenced by the topology of interlayer coupling (i.e., by $\tilde{\mathbf{A}}$), including mathematical and empirical studies of localization. I will explore a few applications including a multiplex airline transportation system and a temporal network that encodes the graduation and hiring of PhDs among mathematics departments in the U.S. (i.e., departmental prestige). Time permitting, I will discuss the closely related topic of multiplex Markov chains [4].

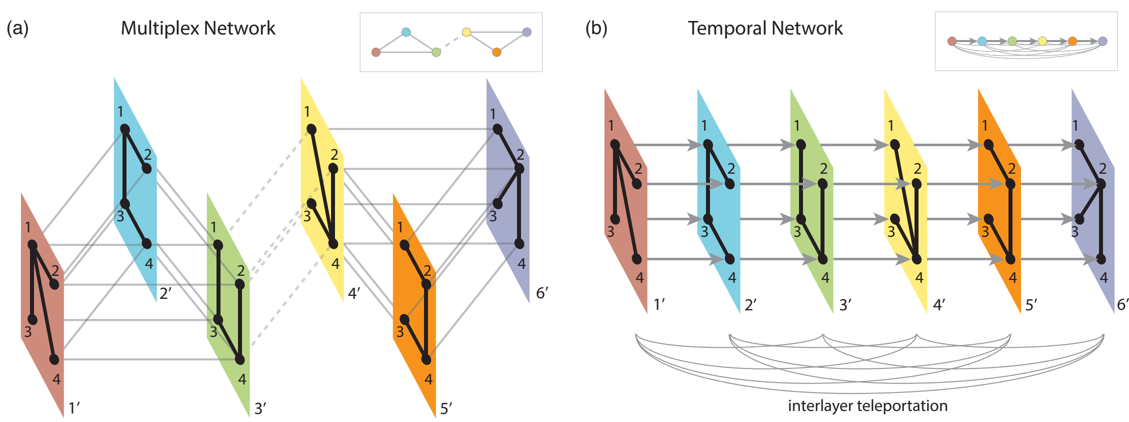

Figure 1: **Schematics of two types of multilayer networks.** (a) A multiplex network in which layers are coupled using an undirected interlayer adjacency matrix $\tilde{\boldsymbol{A}}$. (b) A multiplex representation of a discrete-time temporal network; we couple the sequence of time layers through a directed (time-respecting) chain that has "interlayer teleportation". Formally, these types couplings for multiplex networks are called "diagonal" and "uniform." Note that our framework allows the interlayer couplings to be either undirected or directed. We show how directed coupling between time layers allows us to tunably bias the most important nodes/layers to either be the earliest or the last ones.

References

[1] D. Taylor, S. A. Meyers, A. Clauset, M. A. Porter and P. J. Mucha (2017) Eigenvector-based centrality measures for temporal networks. *Multiscale Modeling and Simulation* 15(1), 537–574.

[2] D. Taylor, M. A. Porter and P. J. Mucha (2019) Supracentrality analysis of temporal networks with directed interlayer coupling. Book Chapter in "Temporal Network Theory", edited by P Holme and J Saramaki. Springer.

[3] D. Taylor, M. A. Porter and P. J. Mucha (2019) Tunable eigenvector-based centralities for multiplex and temporal networks. *arXiv*:1904.02059.

[4] D. Taylor (2020) Multiplex Markov chains. *arXiv*:2004.12820.