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THERE IS ONLY ONE TIME

time and classical equations of motion from quantum entanglement via the Page and Wootters mechanism with generalized coherent states

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unifi & INFN

SUMMARY

* WHAT COMES FIRST

* THIS WORK

SUMMARY

★ WHAT COMES FIRST

i1. is there a problem ?

i2. GCS and the large-N limit

i3. the PAW mechanism

★ THIS WORK

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i1. is there a problem ?

i2. GCS and the large-N limit

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★ THIS WORK

r1. a quantum clock for a quantum system QQ
→ energy-time uncertainty relation

r2. a classical clock for a quantum system CQ
→ Schroedinger equation

r3. a classical clock for a classical system CC
→ Hamilton equations of motion !

IS THERE A PROBLEM ?

- why should there be any

  spacetime is cool

  (and QFT is amazing)

IS THERE A PROBLEM ?

- why should there be any
  spacetime is cool
  (and QFT is amazing)

  ... uhm ...

- time is not a quantum observable

  $\Delta E \Delta t \geq \hbar$ does not make sense

  (and QFT does not help when it comes to
  GR $\longleftrightarrow$ QM)

?

GENERALIZED COHERENT STATES (GCS)

$$\mathcal{H}, \ \mathfrak{g}, \ |G\rangle$$

$$|\Omega\rangle \in \mathcal{H} \longleftrightarrow \Omega \in \mathcal{M}$$

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$$\int_{\mathcal{M}} d\mu(\hat{\Omega})\, |\Omega\rangle\langle\Omega| = \hat{\mathbb{1}}_{\mathcal{H}}\,,\quad d\mu(\hat{\Omega}) \text{ invariant}\,,\quad \langle\Omega|\Omega'\rangle \neq \delta(\Omega - \Omega')$$

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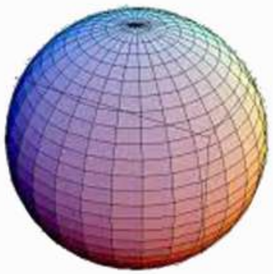
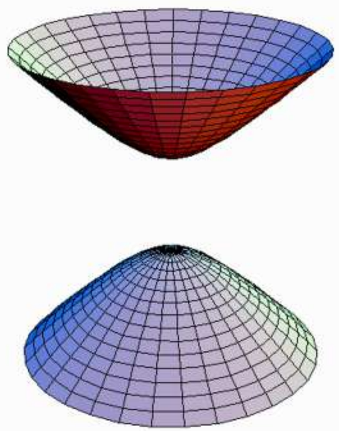
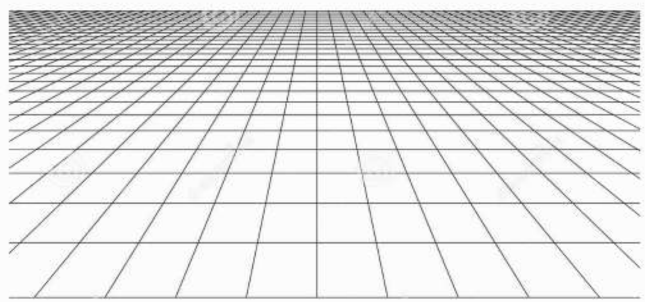
$\mathcal{M}$ is symplectic and depends on $\mathfrak{g}$



$SU(2)$



$SU(1,1)$



h_4

GCS AND THE LARGE-N LIMIT

QUANTUM-TO-CLASSICAL CROSSOVER

conditions : $Q_N \xrightarrow[N\to\infty]{} Cl$, in terms of GCS

GCS AND THE LARGE-N LIMIT

QUANTUM-TO-CLASSICAL CROSSOVER

conditions: $Q_N \xrightarrow[N\to\infty]{} Cl$, in terms of GCS

$\Downarrow$

states $|\psi\rangle$ that survive: $\langle \Omega | \Omega' \rangle \xrightarrow[N\to\infty]{} \delta(\Omega - \Omega')$

$|\Omega\rangle \in \mathcal{H}$ $\quad\quad$ $\Omega \in \mathcal{M}$

observables $\hat{A}$ that survive: $\dfrac{\langle \Omega | \hat{A} | \Omega' \rangle}{\langle \Omega | \Omega' \rangle} \xrightarrow[N\to\infty]{} < \infty$

$\langle \Omega | \hat{A} | \Omega \rangle$ $\quad\quad$ $A(\Omega)$

symbol

GCS AND COMPOSITE SYSTEMS $\Psi = e + \Gamma$

A PARAMETRIC REPRESENTATION

$$|\Psi\rangle\!\rangle = \sum_{\gamma\xi} e_{\gamma\xi} |\xi\rangle \otimes |\gamma\rangle \in \mathcal{H}_\Psi = \mathcal{H}_e \otimes \mathcal{H}_\Gamma$$

GCS AND COMPOSITE SYSTEMS $\Psi = e + \Gamma$

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$$|\Psi\rangle\rangle = \underbrace{\sum_{\gamma\xi} c_{\gamma\xi} |\xi\rangle \otimes |\gamma\rangle}_{\int d\mu(\hat{\Omega})\, |\Omega\rangle\langle\Omega|} \in \mathcal{H}_\Psi = \mathcal{H}_e \otimes \mathcal{H}_\Gamma$$

$$= \int_{\mathcal{M}} d\mu(\Omega)\, \chi(\Omega)\, |\Omega\rangle \otimes |\phi(\Omega)\rangle\rangle$$

GCS AND COMPOSITE SYSTEMS $\Psi = C + \Gamma$

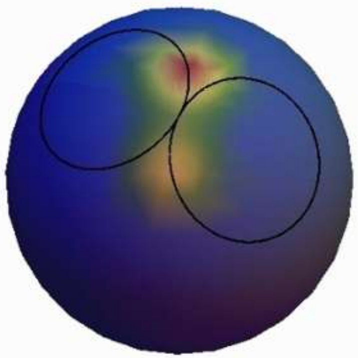
A PARAMETRIC REPRESENTATION

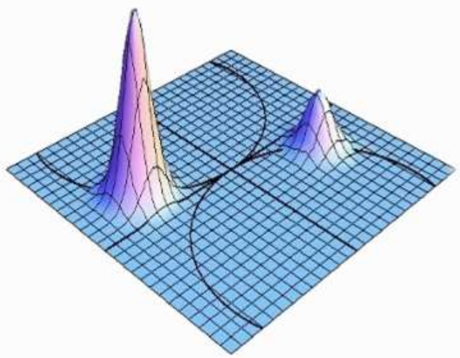
$$|\Psi\rangle\!\rangle = \sum_{\gamma\xi} c_{\gamma\xi} |\xi\rangle \otimes |\gamma\rangle \in \mathcal{H}_\Psi = \mathcal{H}_C \otimes \mathcal{H}_\Gamma$$

$$\underbrace{\qquad\qquad\qquad}_{\int d\mu(\hat{\Omega})|\Omega\rangle\langle\Omega|}$$

$$= \int_{\mathcal{M}} d\mu(\Omega)\, \chi(\Omega) |\Omega\rangle \otimes |\phi(\Omega)\rangle\!\rangle$$

$$\chi^2(\Omega) = \sum_\gamma \Big| \sum_\xi c_{\gamma\xi} \langle \Omega | \xi \rangle \Big|^2$$





THE PAW MECHANISM

@ a certain time t :

conditioned to a clock being in a state labeled by t

THE PAW MECHANISM

@ a certain time t :

conditioned to a clock being in a state labeled by t

↓

in a quantum setting : $\Psi = C + \Gamma$ and 3 assumptions

1. $\hat{H} = \hat{H}_C \otimes \hat{\mathbb{1}}_\Gamma - \hat{\mathbb{1}}_C \otimes \hat{H}_\Gamma$ non interacting

2. $|\Psi\rangle\rangle = \int_{\mathcal{M}} d\mu(\Omega)\, \chi^2(\Omega)\, |\Omega\rangle \otimes |\phi(\Omega)\rangle$ entangled

3. $\hat{H}|\Psi\rangle\rangle = E\,|\Psi\rangle\rangle$ $E = 0$ if you like

GENERALIZED CS + PAW MECHANISM

algebras

hamiltonians

GENERALIZED CS + RAW MECHANISM

algebras

hamiltonians

Cartan decomposition

diagonal & shift

$\hat{D}_\delta$ $\qquad$ $\hat{R}_m$

$$\hat{R}_m^{+} = \hat{R}_{-m}$$

$$[\hat{D}_\delta, \hat{D}_\theta] = 0 \quad ; \quad [\hat{D}_\delta, \hat{R}_m] = d_{\delta m} \hat{R}_m \quad ; \quad [\hat{R}_m, \hat{R}_{-m}] = \sum_\delta d_{\delta m} \hat{D}_\delta \quad ; \quad [\hat{R}_m, \hat{R}_{m'}] = c_{mm'} \hat{R}_{m+m'}$$

GENERALIZED CS + RAW MECHANISM

algebras

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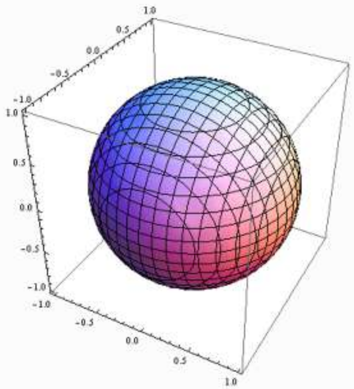
$\hat{D}_\delta$ $\quad\quad$ $\hat{R}_m$

$$\hat{R}_m^{+} = \hat{R}_{-m}$$

$[\hat{D}_\delta, \hat{D}_\theta] = 0$; $[\hat{D}_\delta, \hat{R}_m] = d_{\delta m} \hat{R}_m$; $[\hat{R}_m, \hat{R}_{-m}] = \sum_\delta d_{\delta m} \hat{D}_\delta$; $[\hat{R}_m, \hat{R}_{m'}] = c_{mm'} \hat{R}_{m+m'}$

keep it simple : $\delta = m = 1$ and $d_{1,\pm 1} = \varepsilon \in \mathbb{R}e$ as in su(2)



reference state $|G\rangle : \hat{R}|G\rangle = 0 \Rightarrow \hat{D}|G\rangle \sim |G\rangle$

GENERALIZED CS + RAW MECHANISM

?

algebras

hamiltonians

- Cartan decomposition

  diagonal & shift

$$[\hat{D},\hat{R}] = \varepsilon \hat{R} \qquad [\hat{R},\hat{R}^{+}] = \varepsilon \hat{D}$$

- reference state $\hat{R}|G\rangle = 0$

$$|\lambda\rangle = e^{\lambda \hat{R}^{+} - \lambda^{*}\hat{R}}|G\rangle = N_{\rho} e^{\Lambda \hat{R}^{+}}|G\rangle$$

$$\lambda = \rho e^{-i\varphi} \in \mathbb{C} \qquad \Lambda = |\tan \rho| e^{-i\varphi}$$

GENERALIZED CS + RAW MECHANISM ?

algebras

hamiltonians

- Cartan decomposition

diagonal & shift

$$[\hat{D},\hat{R}] = \varepsilon\hat{R} \qquad [\hat{R},\hat{R}^{+}] = \varepsilon\hat{D}$$

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$$\lambda = \rho e^{-i\varphi} \in \mathbb{C} \qquad \Lambda = |\tan\rho|e^{-i\varphi}$$

$$\hat{H}_c$$

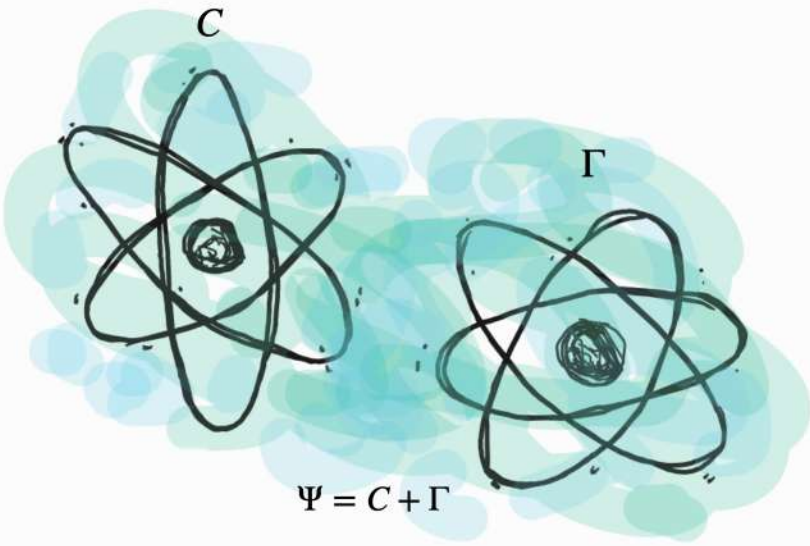
take

$$\hat{D} = \hat{H}_c$$

$$[\hat{H}_c,\hat{R}] = \varepsilon\hat{R}$$

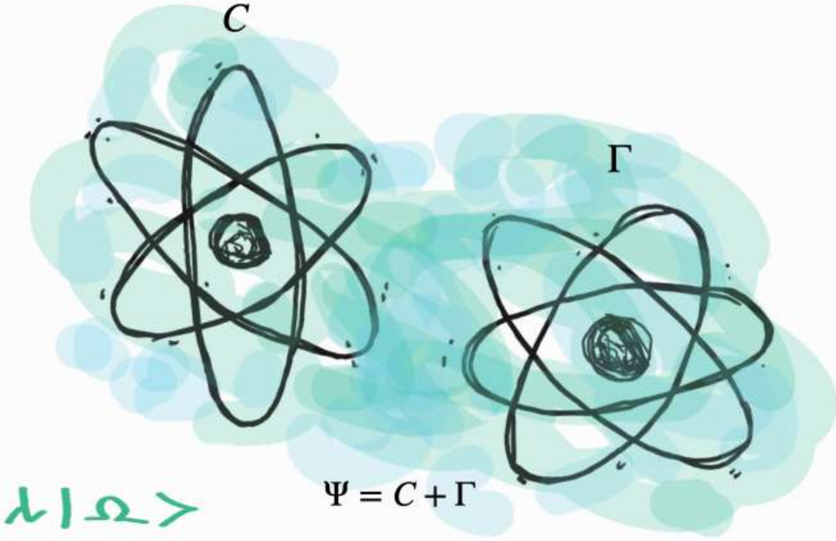
$$\hat{H}_c|G\rangle = \varepsilon_0|G\rangle$$ $= 0$ if you like

A QUANTUM CLOCK FOR A QUANTUM SYSTEM



A QUANTUM CLOCK FOR A QUANTUM SYSTEM

$$\left[\hat{H}_c\,,\ e^{\Lambda^*\hat{R}}\right] = \varepsilon \Lambda^* \hat{R}\, e^{\Lambda^*\hat{R}}$$



$$\langle \lambda | \hat{H}_c | \Omega \rangle = i\varepsilon \frac{d}{d\varphi} \langle \lambda | \Omega \rangle$$

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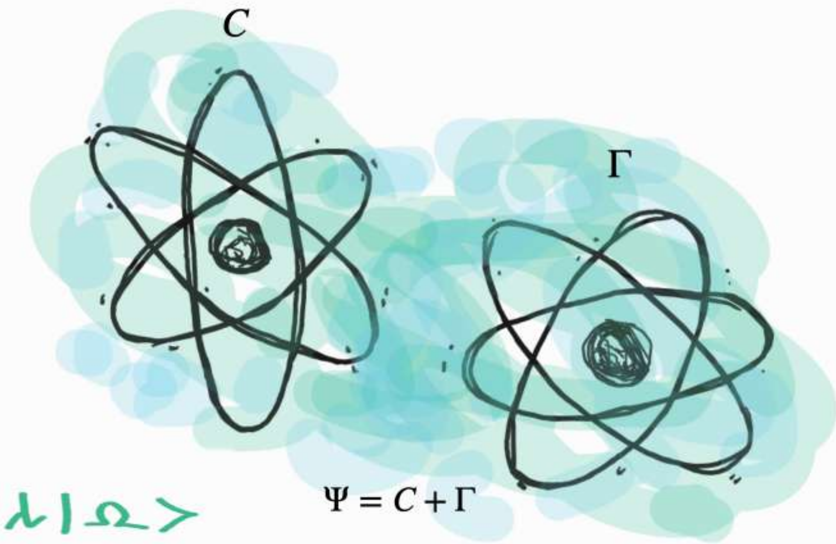

$$\left[\hat{H}_c , e^{\Lambda^* \hat{R}}\right] = \varepsilon \Lambda^* \hat{R} e^{\Lambda^* \hat{R}}$$

$$\langle \lambda | \hat{H}_c | \Omega \rangle = i\varepsilon \frac{d}{d\varphi} \langle \lambda | \Omega \rangle$$

$$+$$

$$\langle \lambda | \hat{H} | \Psi \rangle\rangle = 0$$

$$\downarrow$$

$$i\varepsilon \frac{d}{d\varphi} | \Phi_\varrho(\varphi) \rangle = \hat{H}_\Gamma | \Phi_\varrho(\varphi) \rangle$$

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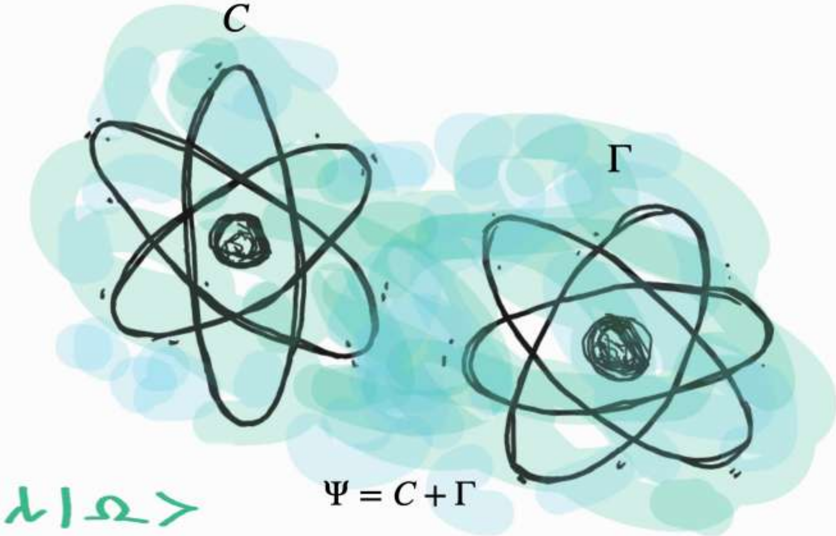

$$[\hat{H}_c , e^{\lambda^*\hat{R}}] = \varepsilon\lambda^*\hat{R}e^{\lambda^*\hat{R}}$$

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$$+$$

$$\langle\lambda|\hat{H}|\Psi\rangle\rangle = 0$$

$$\downarrow$$

$$i\varepsilon\frac{d}{d\varphi}|\Phi_e(\varphi)\rangle = \hat{H}_\Gamma|\Phi_e(\varphi)\rangle$$

Schroed - like

but $|\Phi_e(\varphi)\rangle = \langle\lambda|\Psi\rangle\rangle$

unnormalized

?

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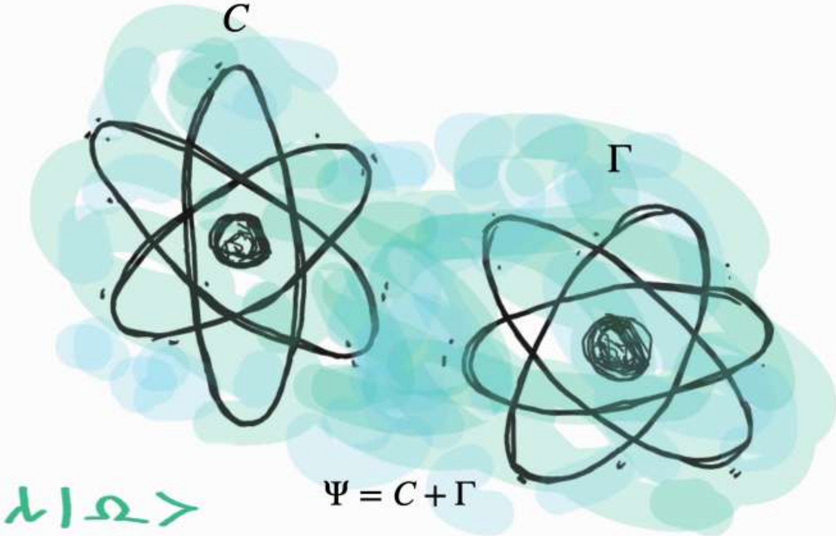

$$[\hat H_c , e^{\Lambda^* \hat R}] = \varepsilon \Lambda^* \hat R e^{\Lambda^* \hat R}$$

$$\langle \lambda | \hat H_c | \Omega \rangle = i\varepsilon \frac{d}{d\varphi} \langle \lambda | \Omega \rangle$$

$+$

$$\langle \lambda | \hat H | \Psi \rangle\rangle = 0$$

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Schroed - like

but $| \Phi_\varrho(\varphi) \rangle = \langle \lambda | \Psi \rangle\rangle$

unnormalized

?

$$\hat R = (\hat R \hat R^\dagger)^{1/2} e^{-i\hat\phi}$$

$$[\hat H_c , \sin\hat\phi] = i\varepsilon \cos\hat\phi$$

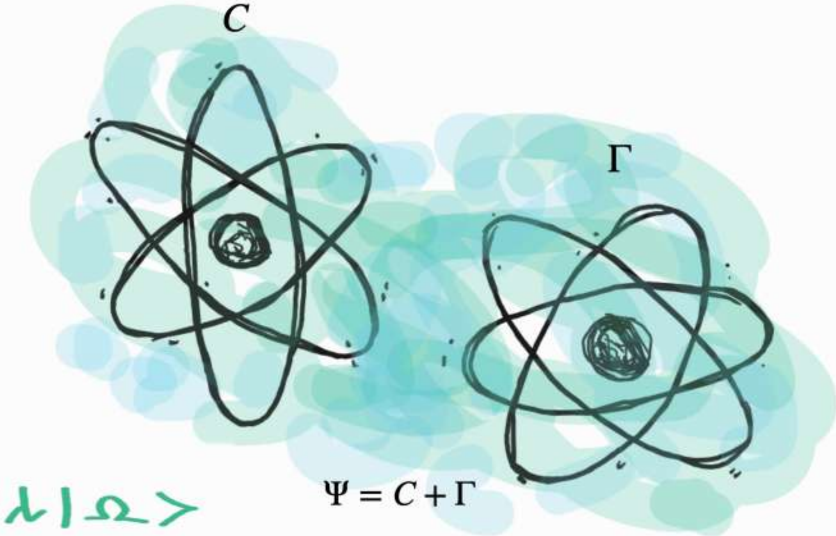
$+$

$$\hat H_c = \hat H + \hat H_\Gamma$$

$\downarrow$

$$\Delta(\hat H + \hat H_\Gamma)\, \Delta \sin\hat\phi \geq \left| \frac{\varepsilon}{2} \langle \cos\hat\phi \rangle \right|$$

A QUANTUM CLOCK FOR A QUANTUM SYSTEM



$$[\hat{H}_c, e^{\Lambda^* \hat{R}}] = \varepsilon \Lambda^* \hat{R} e^{\Lambda^* \hat{R}}$$

$$\langle \lambda | \hat{H}_c | \Omega \rangle = i\varepsilon \frac{d}{d\varphi} \langle \lambda | \Omega \rangle$$

+

$$\langle \lambda | \hat{H} | \Psi \rangle\rangle = 0$$

↓

$$i\varepsilon \frac{d}{d\varphi} | \Phi_\varrho(\varphi) \rangle = \hat{H}_\Gamma | \Phi_\varrho(\varphi) \rangle$$

Schroed-like

but $|\Phi_\varrho(\varphi)\rangle = \langle \lambda | \Psi \rangle\rangle$

unnormalized

?

$$\hat{R} = (\hat{R}\hat{R}^\dagger)^{1/2} e^{-i\hat{\phi}}$$

$$[\hat{H}_c, \sin\hat{\phi}] = i\varepsilon \cos\hat{\phi}$$

+

$$\hat{H}_c = \hat{H} + \hat{H}_\Gamma$$

↓

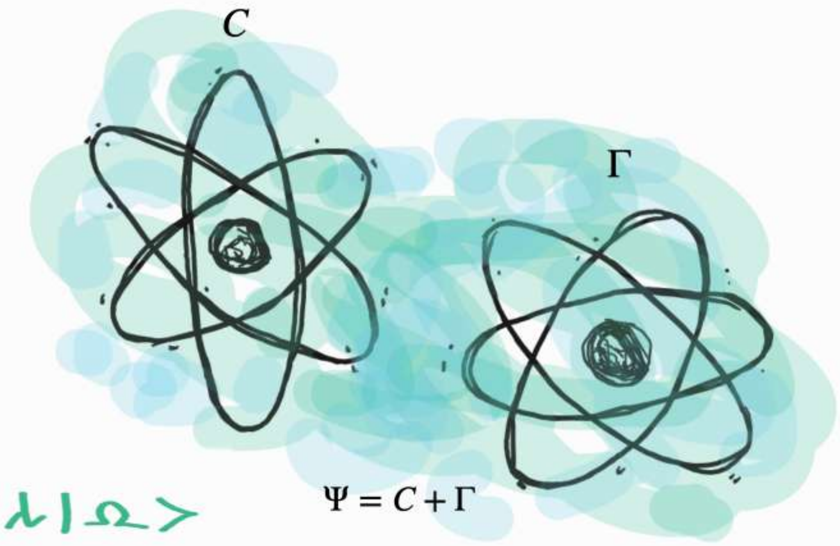
$$\Delta(\hat{H} + \hat{H}_\Gamma)\, \Delta \sin\hat{\phi} \geq \left| \frac{\varepsilon}{2} \langle \cos\hat{\phi} \rangle \right|$$

energy-time-like

but not yet

?

A QUANTUM CLOCK FOR A QUANTUM SYSTEM



$$[\hat{H}_c , e^{\Lambda^*\hat{R}}] = \varepsilon \Lambda^* \hat{R} e^{\Lambda^* \hat{R}}$$

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$$\langle \lambda | \hat{H}_c | \Omega \rangle = i\varepsilon \frac{d}{d\varphi} \langle \lambda | \Omega \rangle$$

$$[\hat{H}_c , \sin\hat{\phi}] = i\varepsilon \cos\hat{\phi}$$

$+$ $\qquad$ $+$

$$\langle \lambda | \hat{H} | \Psi \rangle\rangle = 0$$

$$\hat{H}_c = \hat{H} + \hat{H}_\Gamma$$

$\downarrow$ $\qquad$ $\downarrow$ $\qquad$ $\downarrow$

$$i\varepsilon \frac{d}{d\varphi} | \Phi_\varepsilon(\varphi) \rangle = \hat{H}_\Gamma | \Phi_\varepsilon(\varphi) \rangle$$

$$\Delta(\hat{H} + \hat{H}_\Gamma)\, \Delta \sin\hat{\phi} \geq \left| \frac{\varepsilon}{2} \langle \cos\hat{\phi} \rangle \right|$$

!

Schroed-like

but $| \Phi_\varepsilon(\varphi) \rangle = \langle \lambda | \Psi \rangle\rangle$

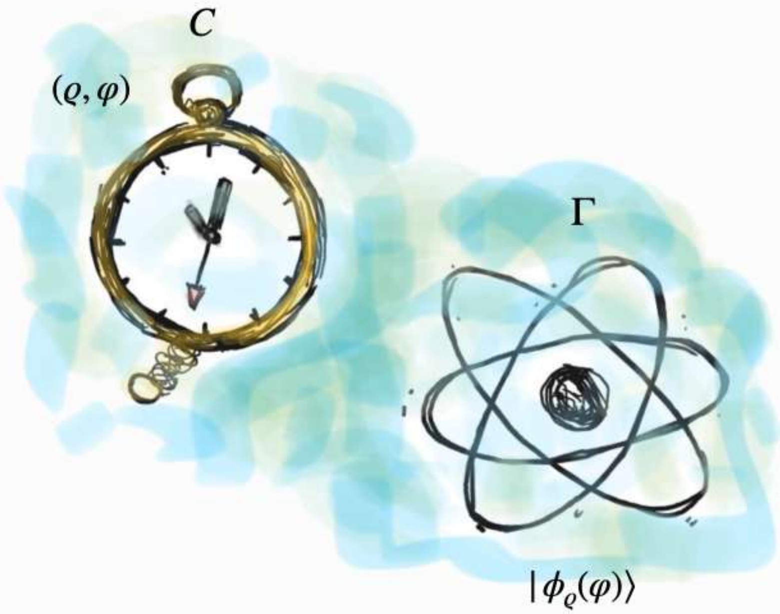
unnormalized

energy-time-like

but not yet

? $\frac{\hbar}{\varepsilon}\varphi$?

A CLASSICAL CLOCK FOR A QUANTUM SYSTEM



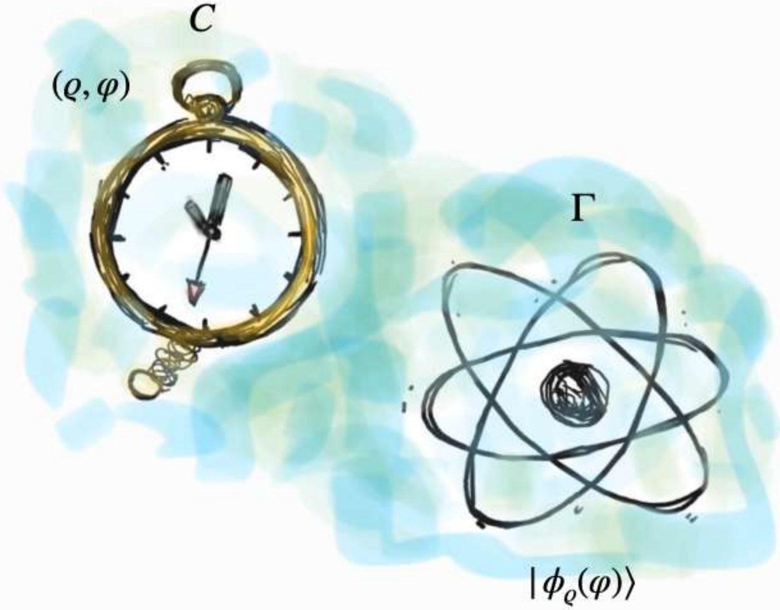
A CLASSICAL CLOCK FOR A QUANTUM SYSTEM

$$\lim_{N\to\infty} \langle \lambda | \Omega \rangle = \delta(\lambda - \Omega)$$

$$\langle \Phi_\varrho(\varphi) | \Phi_\varrho(\varphi) \rangle = \chi^2(\varrho)$$

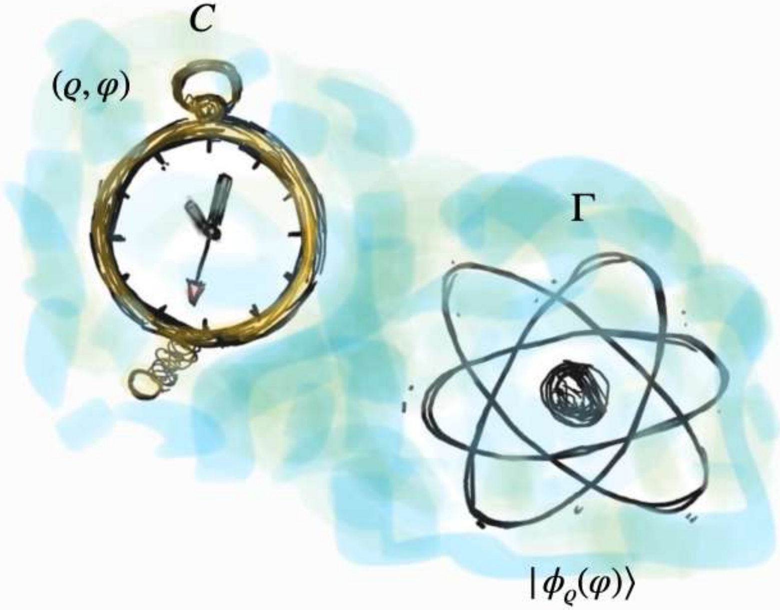
$$|\phi_\varrho(\varphi)\rangle = \frac{|\Phi_\varrho(\varphi)\rangle}{\chi(\varrho)}$$

normalized



$$i\varepsilon \frac{d}{d\varphi} |\phi_\varrho(\varphi)\rangle = \hat{H}_\Omega |\phi_\varrho(\varphi)\rangle$$

A CLASSICAL CLOCK FOR A QUANTUM SYSTEM



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$$\langle\Phi_\varrho(\varphi)|\Phi_\varrho(\varphi)\rangle = \chi^2(\varrho)$$

$$|\phi_\varrho(\varphi)\rangle = \frac{|\Phi_\varrho(\varphi)\rangle}{\chi(\varrho)}$$

normalized

$$\lim_{N\to\infty} \frac{\langle\lambda|\hat{A}|\Omega\rangle}{\langle\lambda|\Omega\rangle} < \infty$$

$$\langle\lambda|\hat{A}|\lambda\rangle = A(\lambda)$$

using

$$\langle\lambda|\hat{H}|\Psi\rangle\rangle = 0$$

from PaW

$$i\varepsilon \frac{d}{d\varphi}|\phi_\varrho(\varphi)\rangle = \hat{H}_\Gamma |\phi_\varrho(\varphi)\rangle$$

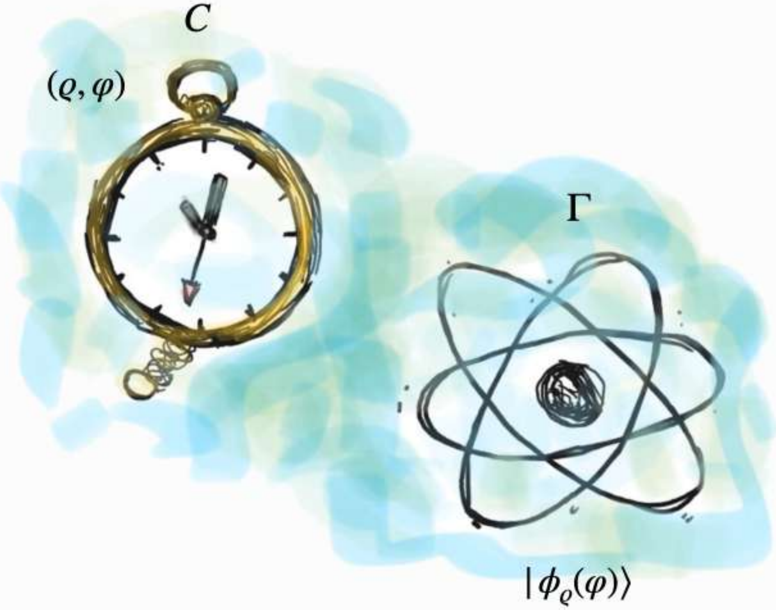
$$\hat{H}_\Gamma |\phi_\varrho(\varphi)\rangle = E_\Gamma(\varrho)|\phi_\varrho(\varphi)\rangle$$

$$E_\Gamma(\varrho) = \langle\lambda|\hat{H}_C|\lambda\rangle$$

$$= \frac{\varepsilon\omega}{2J}\left(1 - \cos 2\varrho\right)$$

$$\Delta E_\Gamma(\varrho)\Delta\varphi \geq \frac{\varepsilon}{2}$$

A CLASSICAL CLOCK FOR A QUANTUM SYSTEM



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normalized

$$\lim_{N\to\infty} \frac{\langle\lambda|\hat{A}|\Omega\rangle}{\langle\lambda|\Omega\rangle} < \infty$$

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$$i\varepsilon \frac{d}{d\varphi}|\phi_\varrho(\varphi)\rangle = \hat{H}_\Gamma |\phi_\varrho(\varphi)\rangle$$

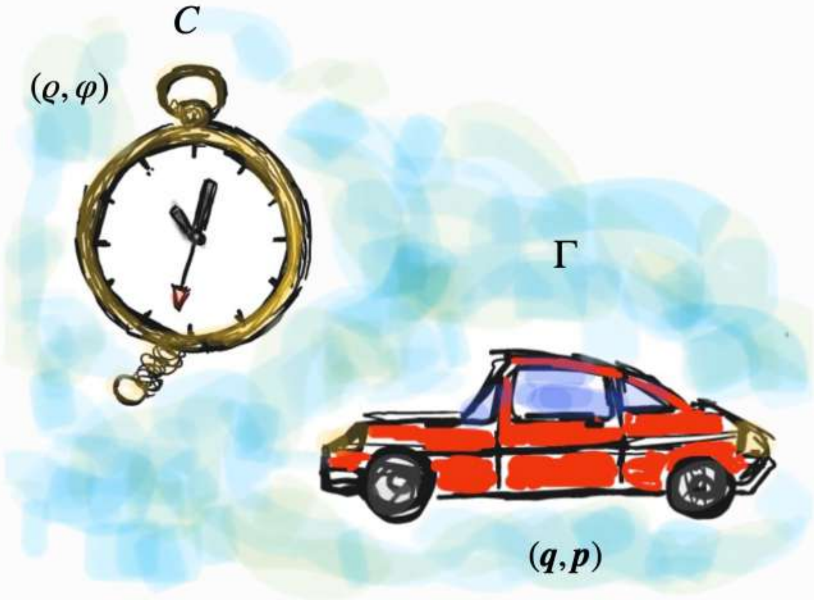
$$\hat{H}_\Gamma |\phi_\varrho(\varphi)\rangle = E_\Gamma(\varrho)|\phi_\varrho(\varphi)\rangle$$

$$E_\Gamma(\varrho) = \langle\lambda|\hat{H}_C|\lambda\rangle = \frac{\varepsilon k}{2J}\left(1 - \cos 2\varrho\right)$$

$$\Delta E_\Gamma(\varrho)\Delta\varphi \geq \frac{\varepsilon}{2}$$

$$t^{QM} = \frac{\hbar}{\varepsilon}\varphi$$

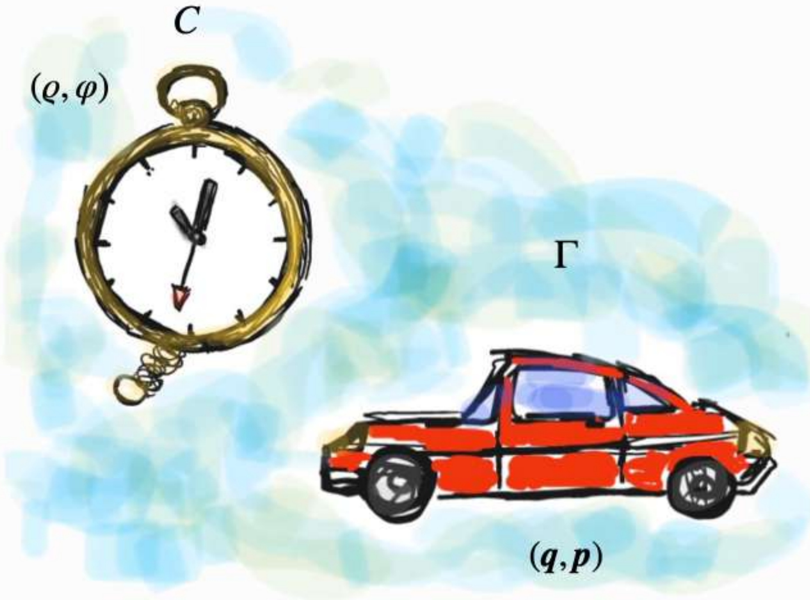
A CLASSICAL CLOCK FOR A CLASSICAL SYSTEM



A CLASSICAL CLOCK FOR A CLASSICAL SYSTEM

GCS $|\Omega\rangle$

$\mathcal{M}_c$

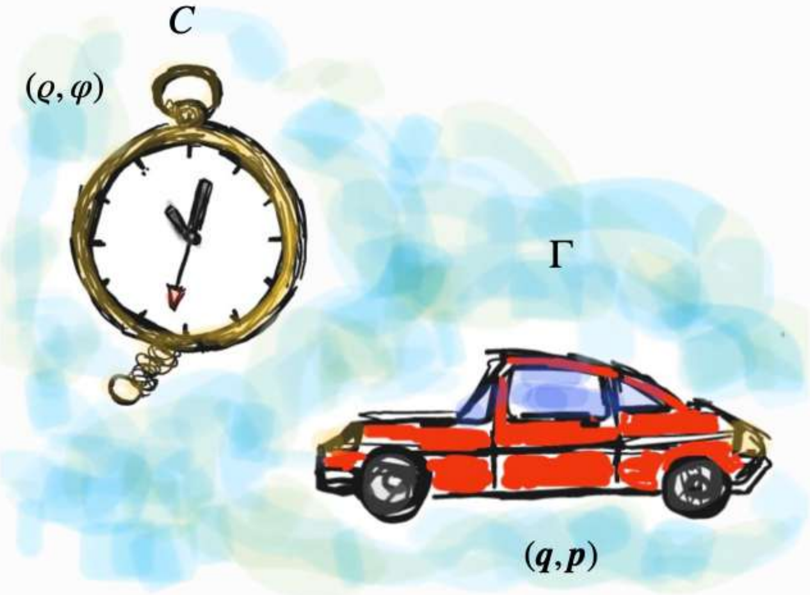


GCS $|\gamma\rangle$

$\mathcal{M}_\Gamma$

$$|\Psi\rangle\!\rangle = \int_{\mathcal{M}_c} d\mu(\Omega) \int_{\mathcal{M}_\Gamma} d\mu(\gamma)\, \beta(\Omega, \gamma)\, |\Omega\rangle \otimes |\gamma\rangle$$

A CLASSICAL CLOCK FOR A CLASSICAL SYSTEM



GCS $|\Omega\rangle$ $\mathcal{H}_c$

GCS $|\gamma\rangle$ $\mathcal{H}_S$

$$|\Psi\rangle\!\rangle = \int_{\mathcal{H}_c} d\mu(\Omega) \int_{\mathcal{H}_S} d\mu(\gamma)\, \beta(\Omega,\gamma)\, |\Omega\rangle \otimes |\gamma\rangle$$

$$+$$

using $\hat{H}|\Psi\rangle\!\rangle = 0$ from PaW

$$\downarrow$$

$\langle\Omega|H_c|\Omega\rangle = \langle\gamma|H_S|\gamma\rangle$ for Ω, γ : $\beta(\Omega,\gamma) \neq 0$

A CLASSICAL CLOCK FOR A CLASSICAL SYSTEM

$\Omega, \gamma : \beta(\Omega, \gamma) \neq 0$

C

(ϱ, φ)

λ, μ

$\hookrightarrow \quad \beta(\lambda, \mu) \neq 0$

Γ

$\lambda = \varrho e^{-i\varphi}$

$H_c(\varrho) = H_s(\mu)$ (q, p)

A CLASSICAL CLOCK FOR A CLASSICAL SYSTEM

$\Omega, \gamma : \beta(\Omega, \gamma) \neq 0$

C

(ϱ, φ)

λ, μ

$\longmapsto$ $\beta(\lambda, \mu) \neq 0$

Γ

$\lambda = \rho e^{-i\varphi}$

$H_c(\rho) = H_s(\mu)$ (q,p)

$\downarrow$

F

$:$

$H_c(\rho) = H_s(\mu = F(\lambda))$

A CLASSICAL CLOCK FOR A CLASSICAL SYSTEM

$\Omega, \gamma : \mathcal{B}(\Omega, \gamma) \neq 0$

C

(ϱ, φ)

λ, μ

$\hookrightarrow \quad \mathcal{B}(\lambda, \mu) \neq 0$

Γ

$\lambda = \varrho e^{-i\varphi}$

$\mathcal{M}_S$ is symplectic

$\exists\, \mathcal{D} : \gamma \to q, p \ /$

$\{q, p\}_S = \frac{1}{\hbar}$

for this pair choose eq

$q - ip = \sqrt{2E}\, \sin \varrho\, e^{-i\varphi}$

$H_C(\varrho) = H_S(\mu)$ (q, p)

$\downarrow$

F

$:$

$H_C(\varrho) = H_S(\mu = F(\lambda))$

?

A CLASSICAL CLOCK FOR A CLASSICAL SYSTEM

$\Omega, \gamma : \beta(\Omega, \gamma) \neq 0$

λ, μ

$\hookrightarrow \quad \beta(\lambda, \mu) \neq 0$

$\lambda = \varrho e^{-i\varphi}$

$\mathcal{M}_S$ is symplectic

$\exists\, \mathcal{D} : \gamma \to q, p$ /

$\{q, p\}_S = \frac{1}{\hbar}$

for this pair choose eq

$q - ip = \sqrt{k\varepsilon}\, \sin\varrho\, e^{-i\varphi}$

$$H_c(\varrho) = H_S(\mu)$$

$$\downarrow$$

$$F$$

$$:$$

?

$$\frac{k\varepsilon}{2}(\cos 2\varrho - 1) = H_c(\varrho) = H_S(\mu = F(\lambda)) = k\varepsilon(q^2 + p^2)$$

A CLASSICAL CLOCK FOR A CLASSICAL SYSTEM

$\Omega, \gamma : \beta(\Omega, \gamma) \neq 0$

$\lambda, \mu \hookrightarrow$

$$\beta(\lambda, \mu) \neq 0$$

$\lambda = \varrho e^{-i\varphi}$

M_Γ is symplectic

$\exists\, \mathfrak{b} : \gamma \to q, p$ /

$\{q, p\}_\Gamma = \frac{1}{\hbar}$

for this pair choose eq

$q - ip = \sqrt{k\varepsilon}\, \sin\varrho\, e^{-i\varphi}$

$H_c(\varrho) = H_\Gamma(\mu)$ (q,p)

$\downarrow$

F

$:$

$\frac{k\varepsilon}{2}(\cos 2\varrho - 1) = \quad H_c(\varrho) = H_\Gamma(\mu = F(\lambda)) \quad = k\varepsilon(q^2 + p^2)$

?

$\downarrow$

F^*

pullback-by-F

A CLASSICAL CLOCK FOR A CLASSICAL SYSTEM

$\Omega, \gamma : \beta(\Omega, \gamma) \neq 0$

λ, μ

$\hookrightarrow$

C

(ϱ, φ)

$$\beta(\lambda, \mu) \neq 0$$

Γ

$\lambda = \varrho e^{-i\varphi}$

$\mathcal{M}_\Gamma$ is symplectic

$\exists\, \phi : \gamma \to q, p$ /

$\{q, p\}_\Gamma = \frac{1}{\hbar}$

for this pair

choose eq

$q - ip = \sqrt{k\varepsilon}\, \sin\varrho\, e^{-i\varphi}$

$$H_C(\varrho) = H_\Gamma(\mu) \quad (q,p)$$

$$\downarrow$$

$$F$$

$$\vdots$$

$$\frac{k\varepsilon}{2}(\cos 2\varrho - 1) = H_C(\varrho) = H_\Gamma(\mu = F(\lambda)) = k\varepsilon(q^2 + p^2)$$

?

$$\downarrow$$

$$F^*$$

pullback-by-F

$\{q, p\}_\Gamma \xrightarrow[F^*]{} \{f, g\}_C$ write $\{q, H_C\}_C \xrightarrow[F]{} \{q, H_\Gamma\}_\Gamma$

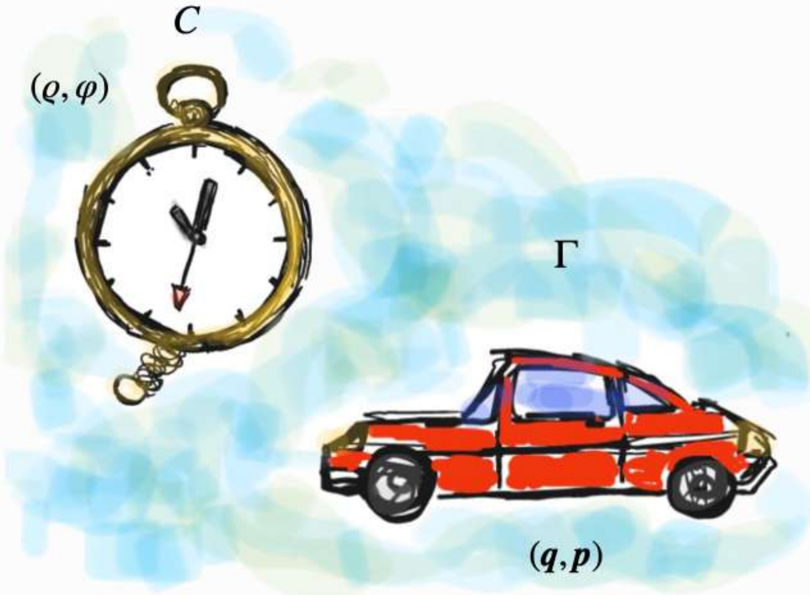
$= (\hbar\varepsilon \sin(2\varrho))^{-1}$

$\times$

$\frac{\partial f}{\partial \varrho}\frac{\partial g}{\partial \varphi} - \frac{\partial g}{\partial \varrho}\frac{\partial f}{\partial \varphi}$

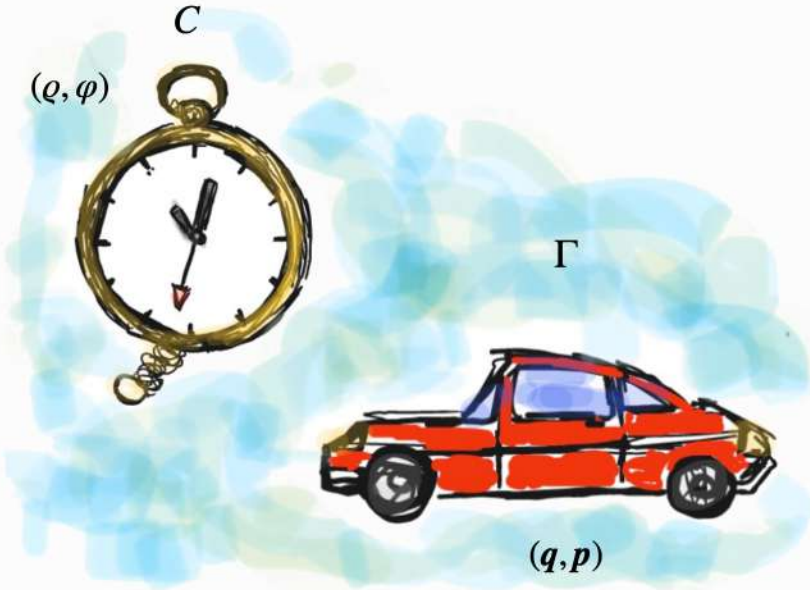
and finally find

A CLASSICAL CLOCK FOR A CLASSICAL SYSTEM



$$\{q, H_\Gamma\}_\Gamma = \frac{\varepsilon}{\hbar}\frac{dq}{d\varphi} \qquad \{p, H_\Gamma\}_\Gamma = \frac{\varepsilon}{\hbar}\frac{dp}{d\varphi}$$

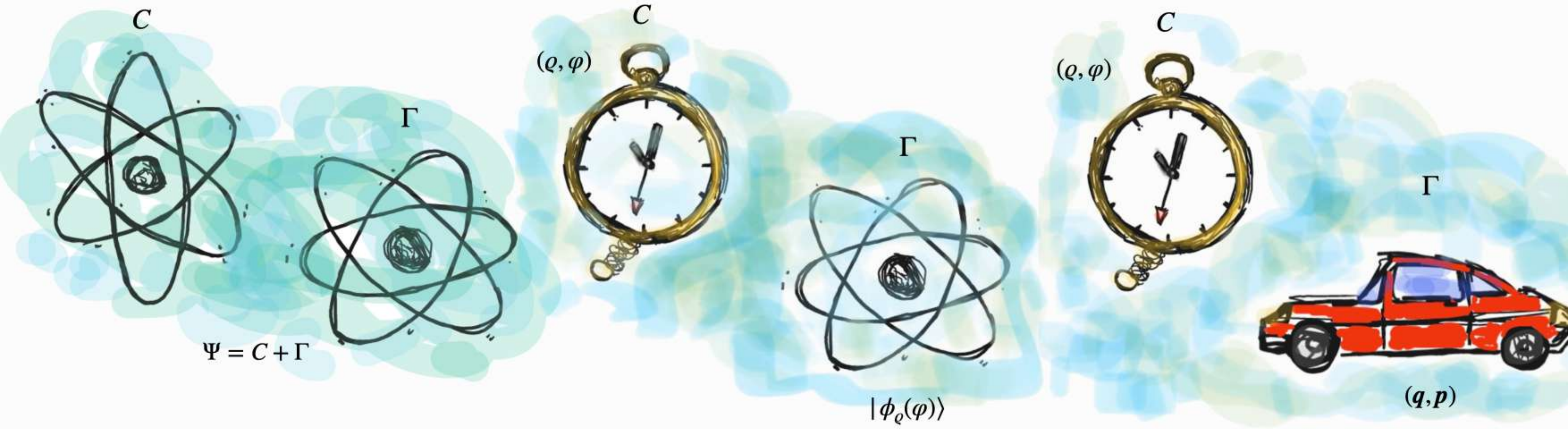
A CLASSICAL CLOCK FOR A CLASSICAL SYSTEM



$$\{q, H_\Gamma\}_\Gamma = \frac{\varepsilon}{\hbar}\frac{dq}{d\varphi} \qquad \{p, H_\Gamma\}_\Gamma = \frac{\varepsilon}{\hbar}\frac{dp}{d\varphi}$$

setting $\hbar = k$

$$t^{CL} = \frac{k}{\varepsilon}\varphi$$



$$\frac{\hbar}{\varepsilon}\varphi \quad = \quad t^{QM} \quad = \quad t^{cl}$$

THERE IS ONLY ONE TIME

and it is a manifestation of entanglement

- energy-time uncertainty relation
- $\beta(\lambda,\mu) = \beta(\varrho,\varphi;q,p) \rightarrow \beta(E,\varphi;q,p) \rightarrow$ spacetime ...
- Schroedinger and geodesic . . .

Refer to

nature COMMUNICATIONS

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Check for updates

Time and classical equations of motion from quantum entanglement via the Page and Wootters mechanism with generalized coherent states

Caterina Foti[1,2,3✉], Alessandro Coppo[1,2], Giulio Barni[1], Alessandro Cuccoli[1,2] & Paola Verrucchi[1,2,4]

and

PAW # 1, 4, 5

GCS # 23, 25, 40

LARGE-N # 19, 21

PARAMETRIC REPRESENTATION WITH GCS # 24, 41