# Critical Parametric Quantum Sensing

Simone Felicetti

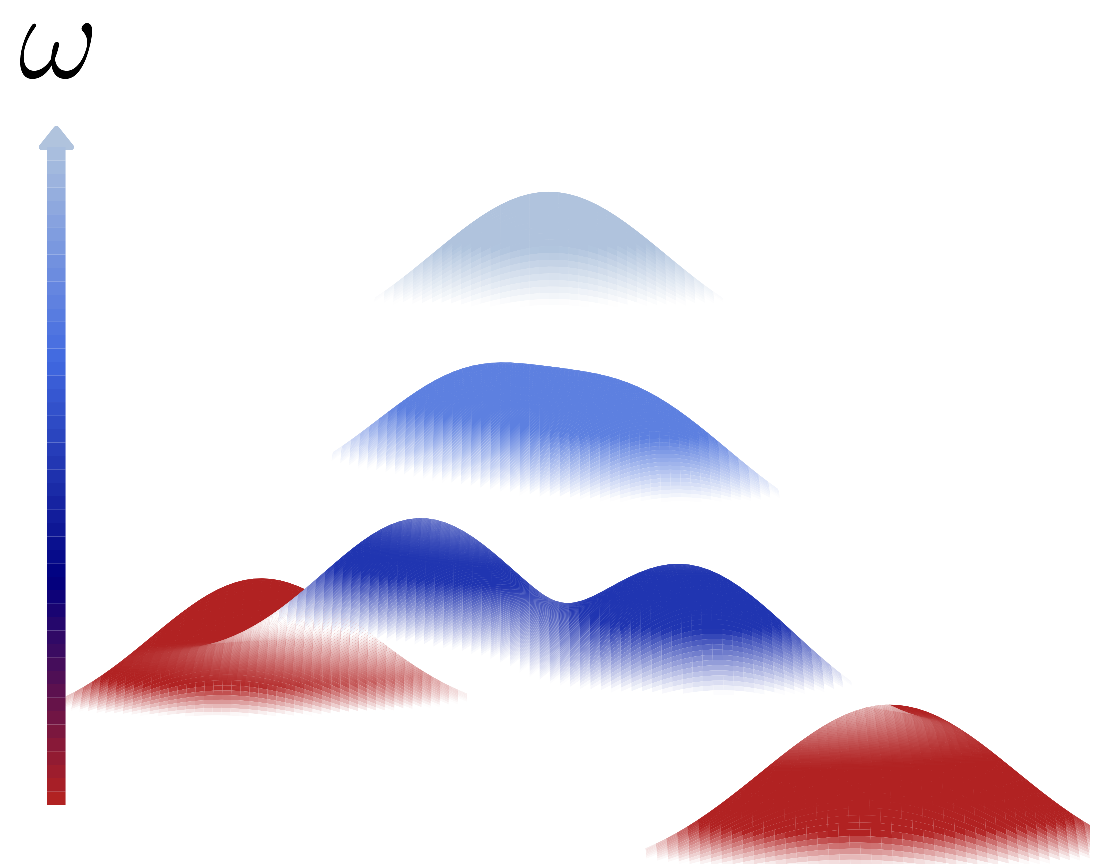
*Istituto di Fotonica e Nanotecnologie CNR-IFN, Rome*

21/10/2021

# Outline



**Finite-component phase transitions** ⟷ **Quantum metrology**



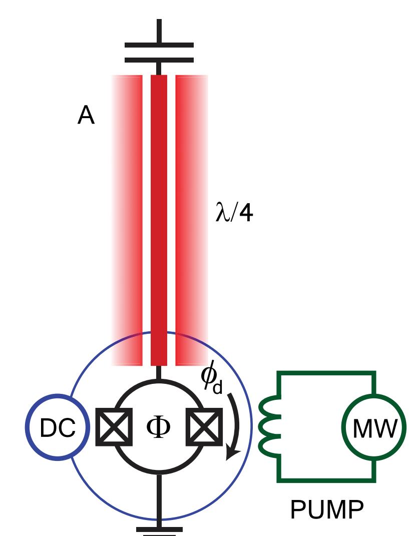
# Outline



**Finite-component phase transitions** ⟷ **Quantum metrology**

- **Fundamental limits** Resources: $\langle \hat{N} \rangle$ 

- **Practical applications**
  1- Quantum Magnetometry
  2- Qubit readout

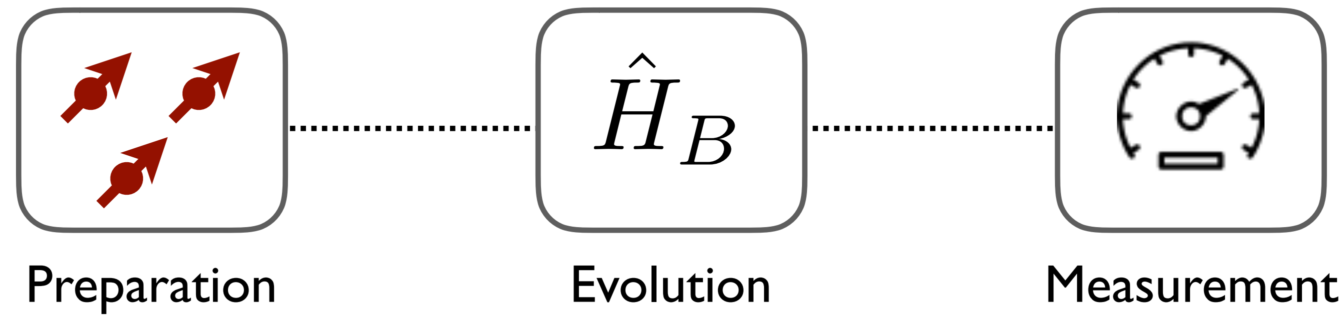


# Introduction



## Quantum metrology

$$\hat{H}_B = \hat{H}_{sys} + B\ \hat{H}_I$$



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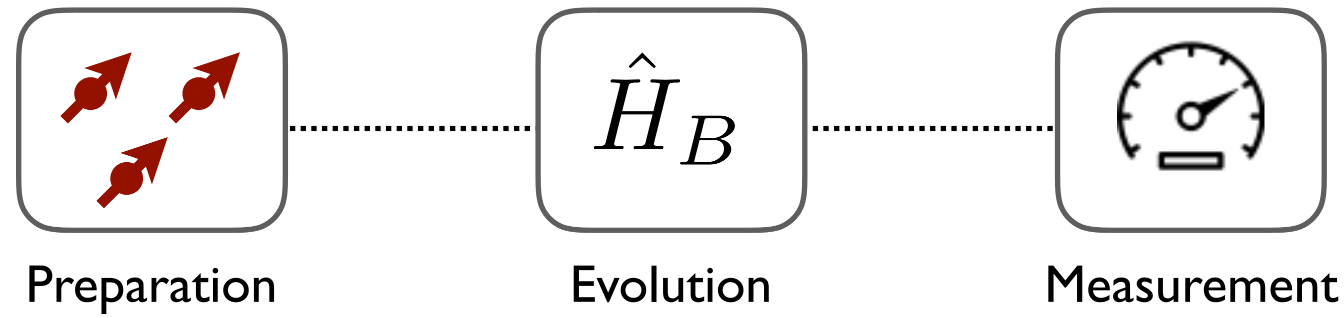


Estimation error: $\delta B = \dfrac{1}{\sqrt{G_B}}$

↓ Quantum Fisher Information

**Classical probes**

$$G_B \sim N$$

**Quantum probes**

$$G_B \sim N^2$$

Heisenberg limit

# Introduction



## Quantum metrology

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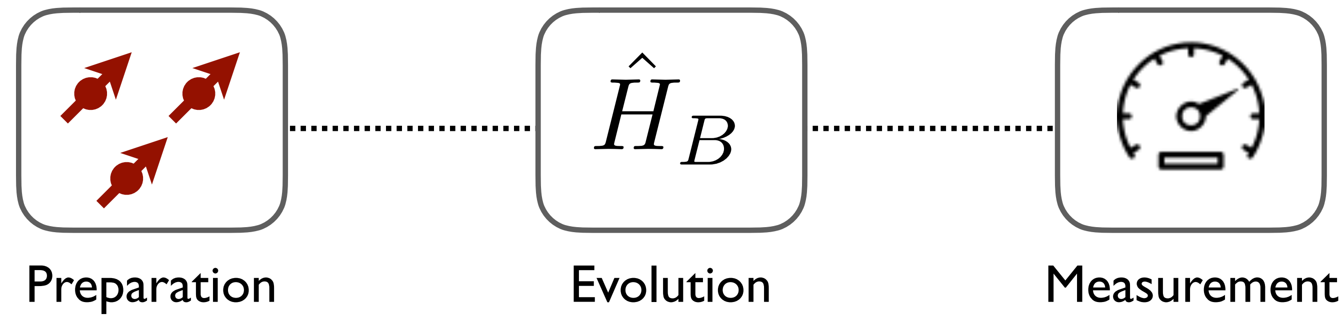


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**Quantum probes**

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Heisenberg limit

**Critical quantum metrology:** $\hat{H}_{sys}$ → **Quantum phase transition**

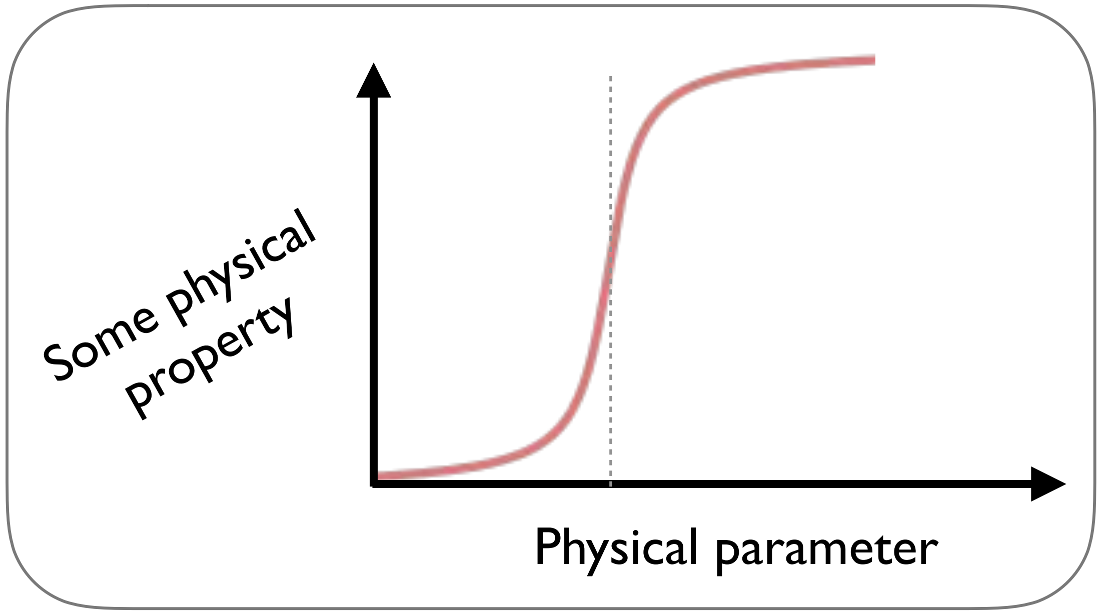
P. Zanardi et al., Phys. Rev. A **78**, 042105 (2008)

K. Macieszczak et al., Phys. Rev. A **93**, 022103 (2016)

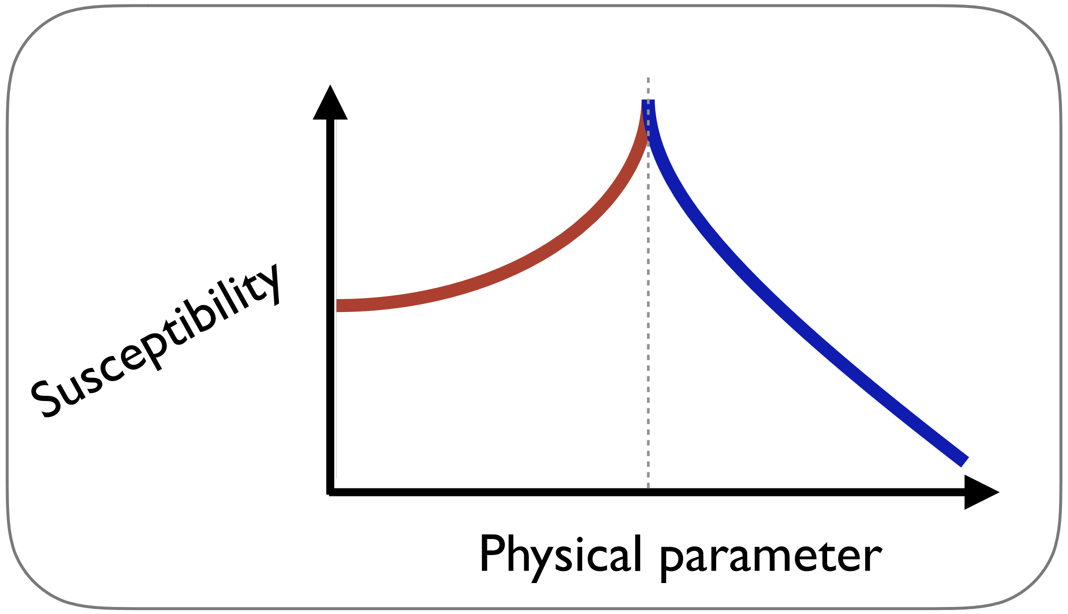
# Introduction



## Critical phase transition



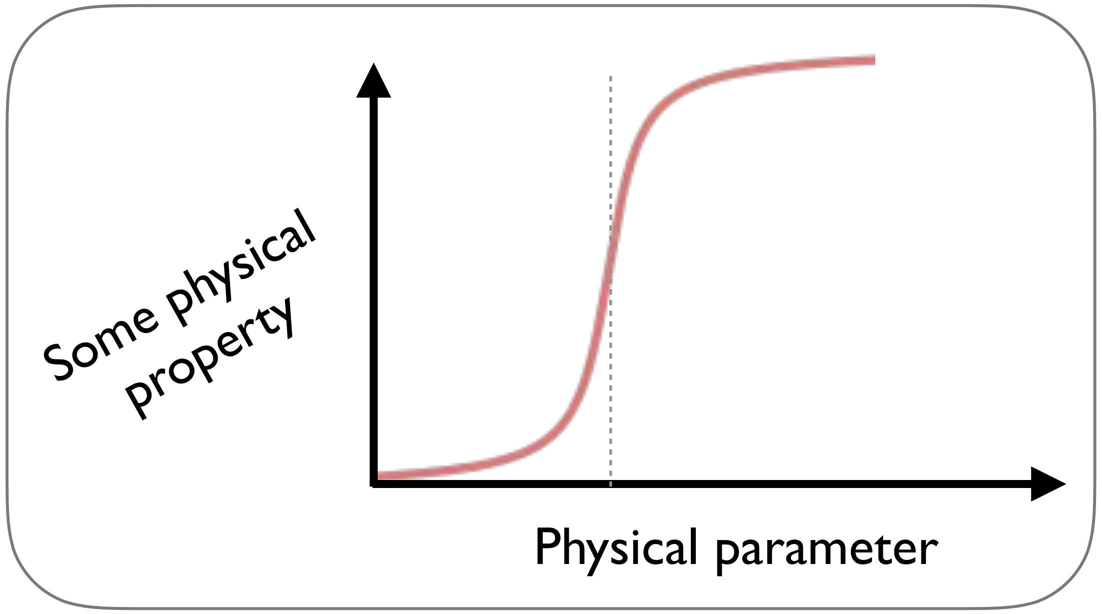

## High sensitivity



# Introduction



## Critical phase transition

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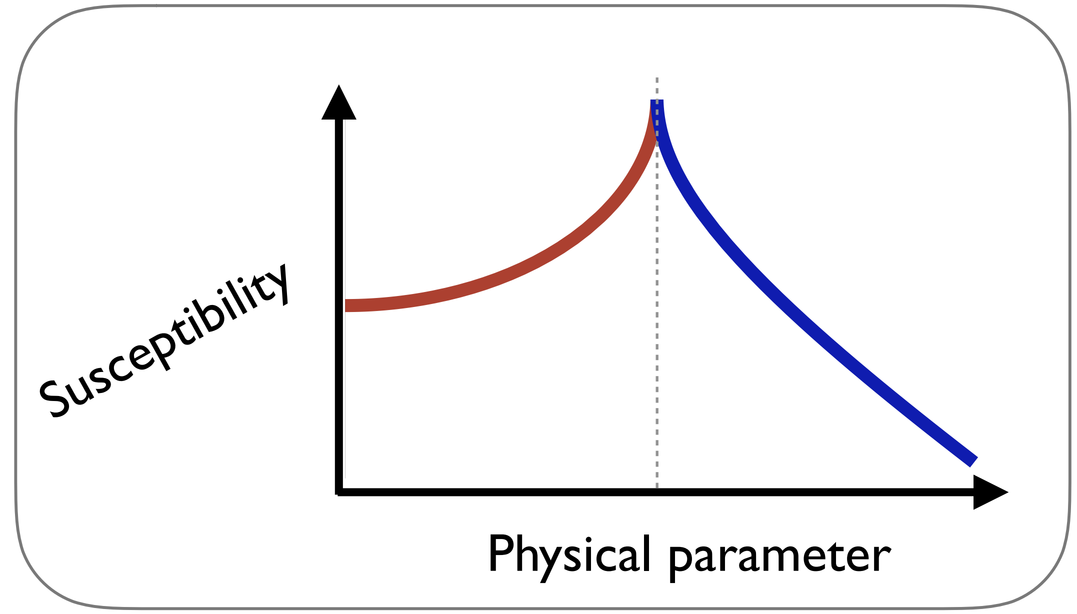



## Critical sensors

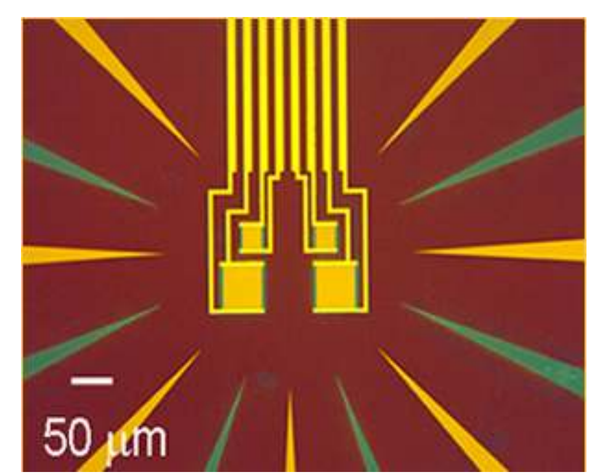
### Bubble chamber

(Liquid-gas)

(CERN image archives)

### Transition-edge sensors

(Superconductor-normal)

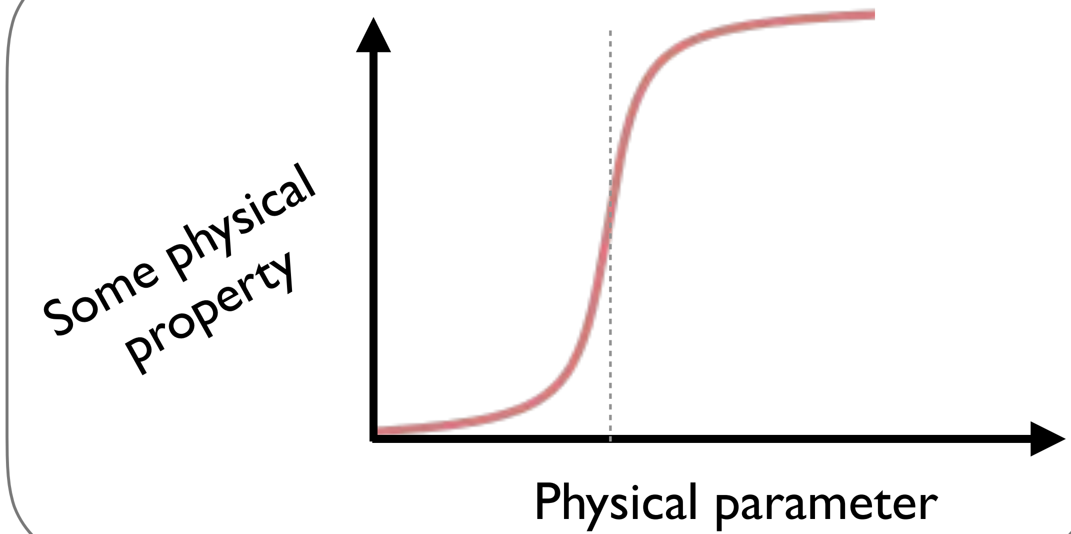



(NIST image archives)

# Introduction



### Critical phase transition

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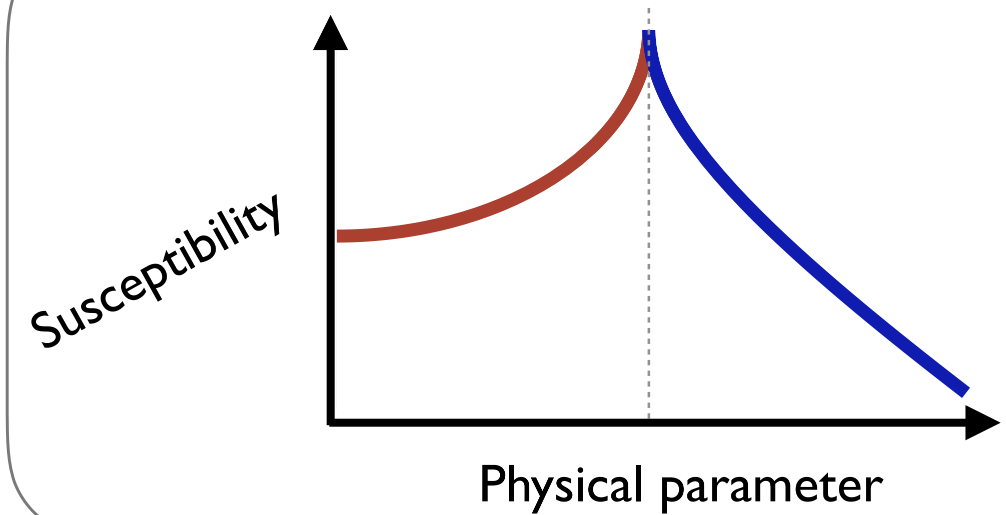
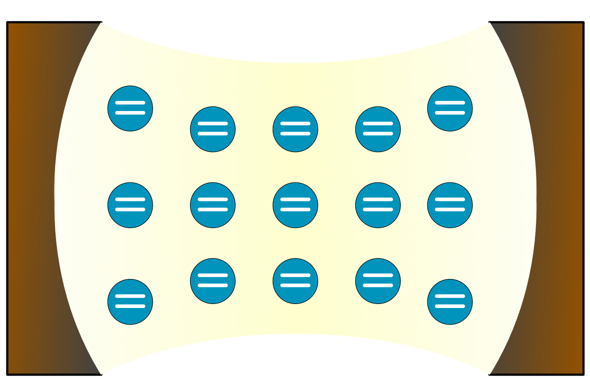



## Critical quantum sensors

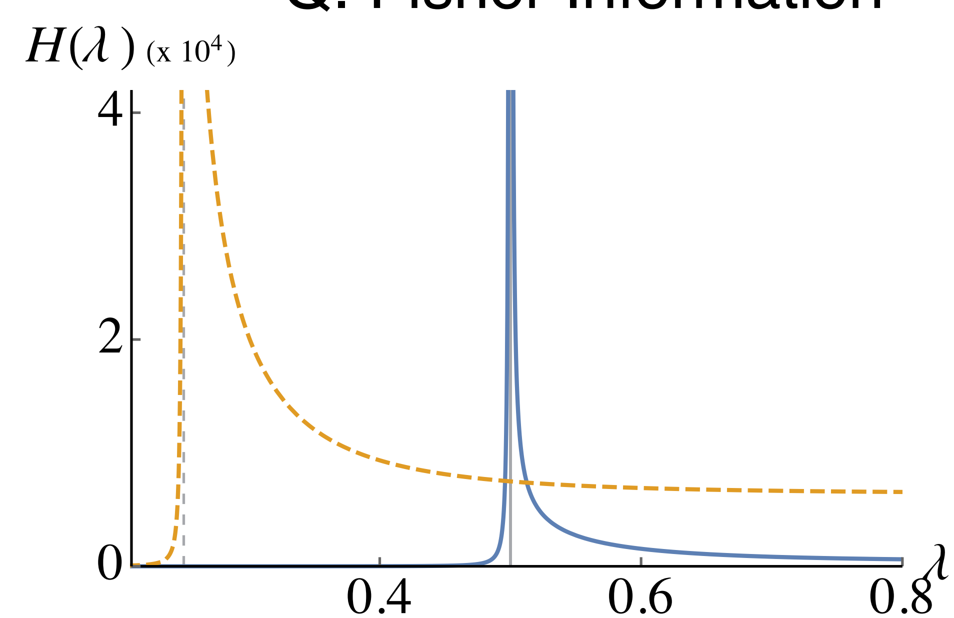
### Dicke model

Superradiant phase transition



$N \to \infty$

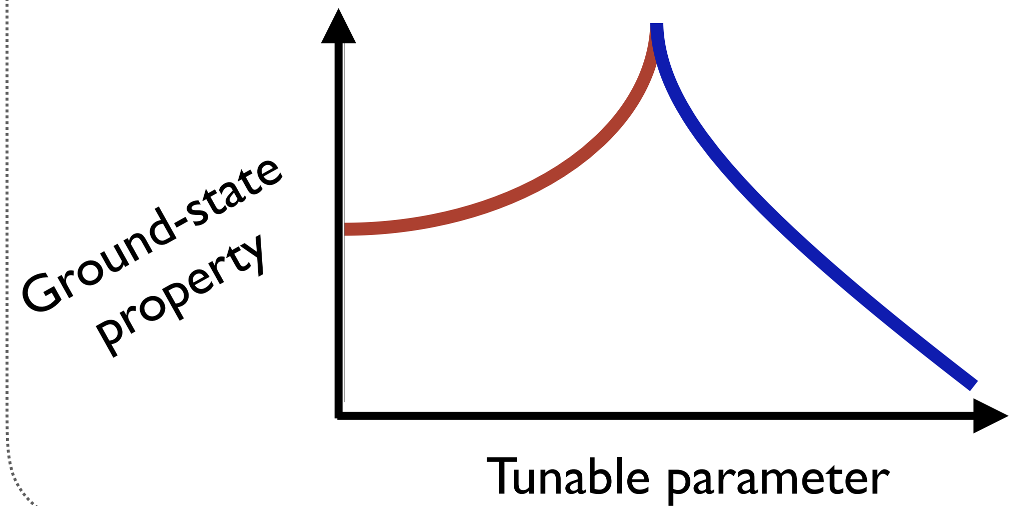
Q. Fisher Information




- M. Bina et al., Phys. Rev. E 93, 052118 (2016).

# Introduction

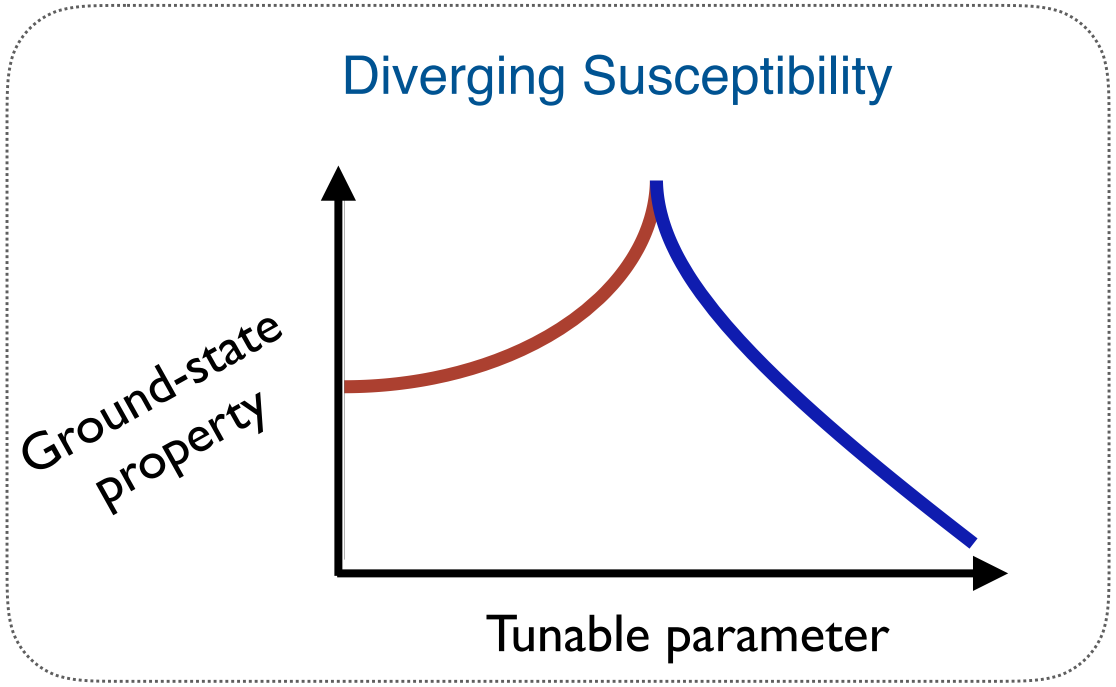
Diverging Susceptibility

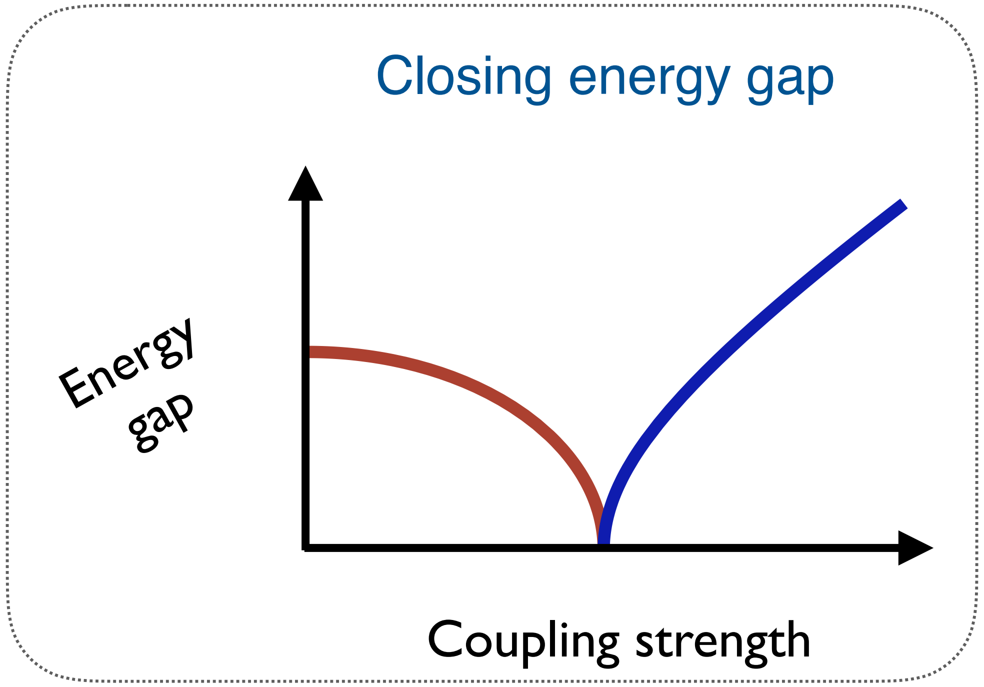



**Infinite precision**

# Introduction





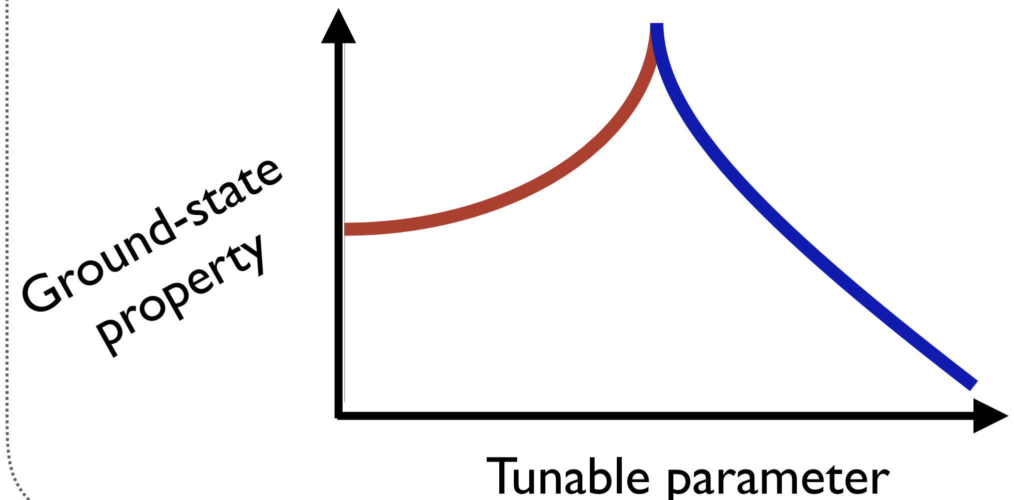

**Infinite precision**

**Infinite time..**

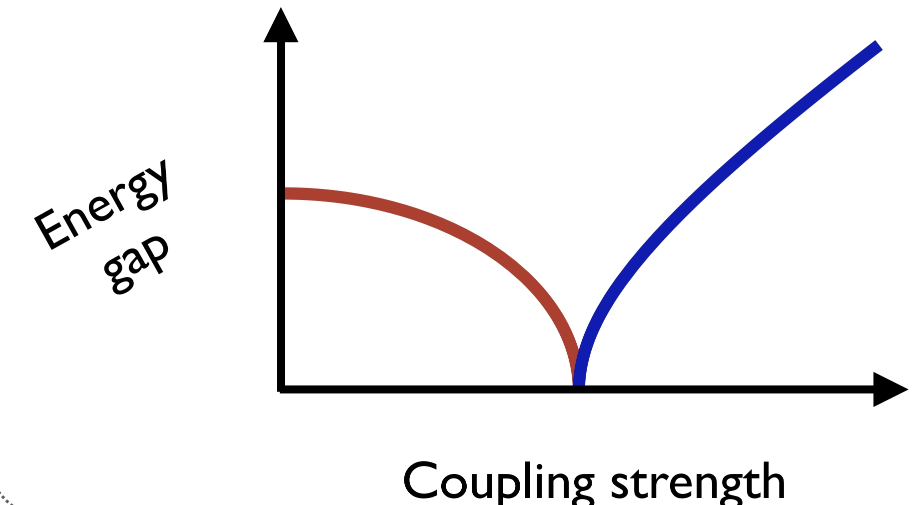
# Introduction



Diverging Susceptibility




Closing energy gap



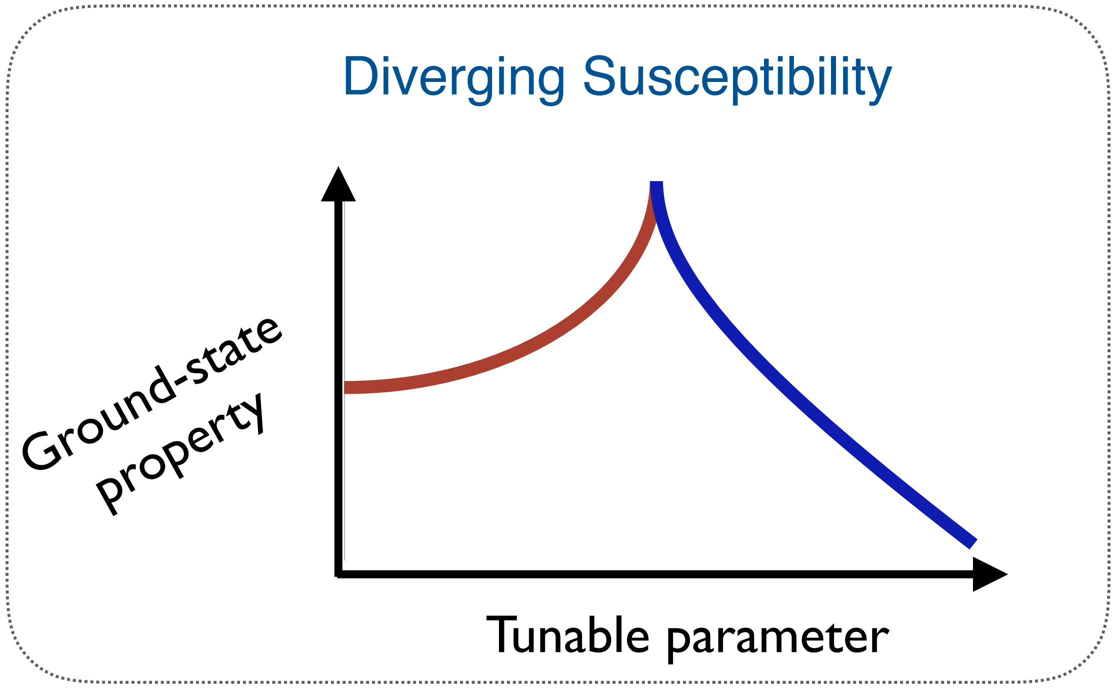

**Infinite precision**

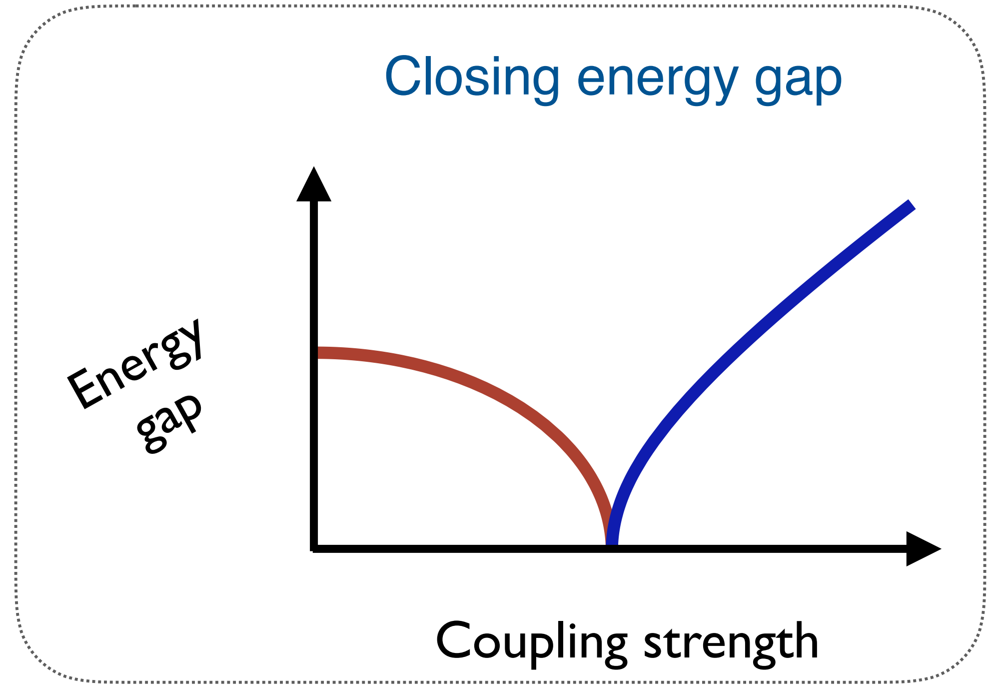
**Infinite time..**

**Time is a resource**
**CQM**

# Introduction



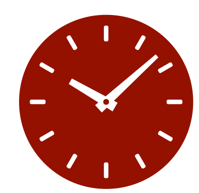



**Infinite precision**

**Infinite time..**

**Time is a resource**
**CQM**

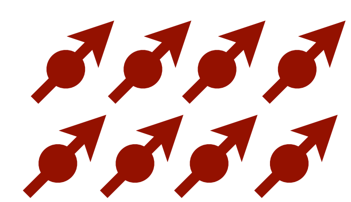


PHYSICAL REVIEW X **8,** 021022 (2018)

**At the Limits of Criticality-Based Quantum Metrology: Apparent Super-Heisenberg Scaling Revisited**

Marek M. Rams,[1,*] Piotr Sierant,[1,†] Omyoti Dutta,[1,2] Paweł Horodecki,[3,‡] and Jakub Zakrzewski[1,4,§]

## For spins

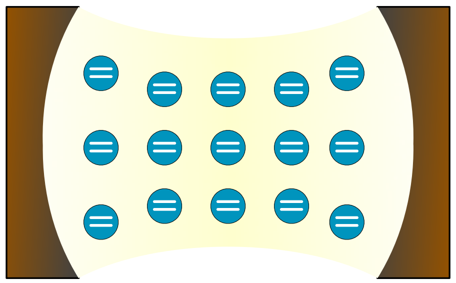


$$G_B \sim t^2 N^2$$

# Introduction



**Dicke model**

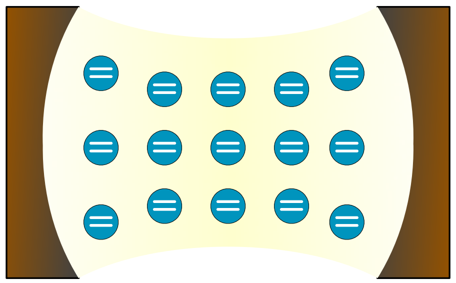


$$H = \omega_c a^\dagger a + \frac{\omega_q}{2}\sum_{i=1}^{N}\sigma_i^z + \frac{g}{\sqrt{N}}(a + a^\dagger)\sum_{i=1}^{N}\sigma_i^x$$

# Introduction



**Dicke model**



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**Thermodynamic limit** $N \to \infty$

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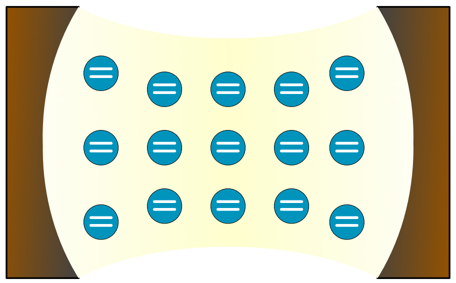

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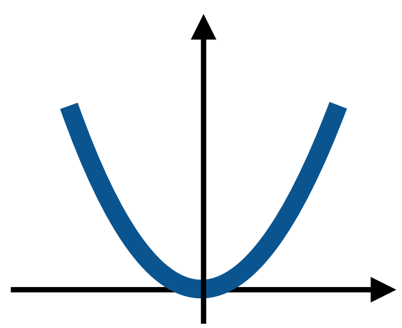
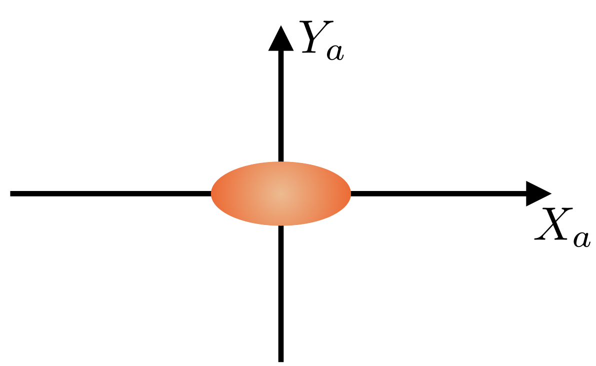
Normal Phase $g < \frac{\sqrt{\omega_c\omega_q}}{2}$

Harmonic potential



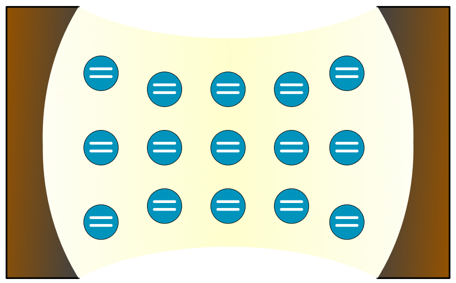
(Squeezed) vacuum



# Introduction



## Dicke model



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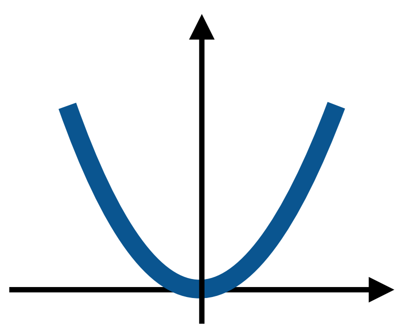

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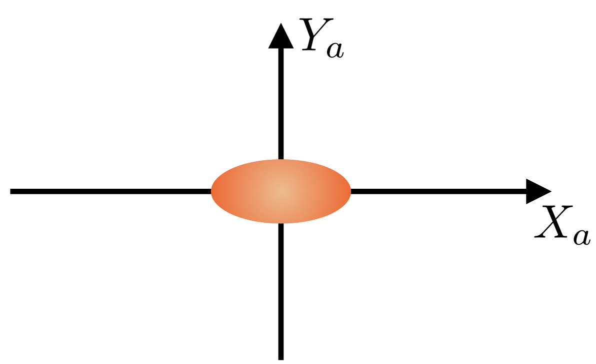
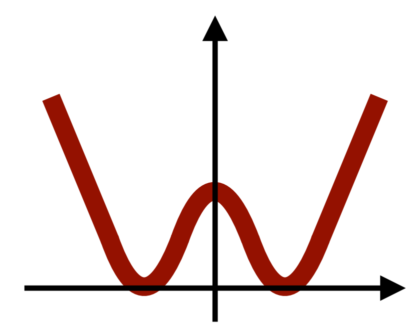
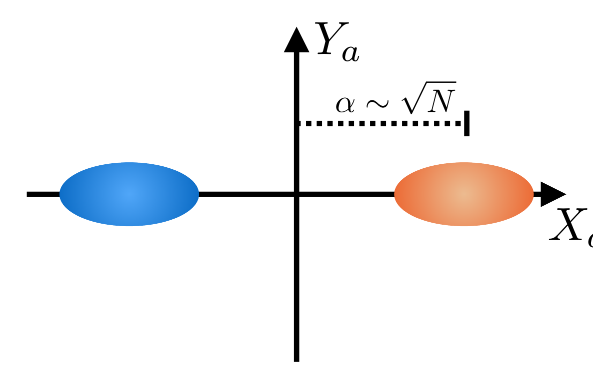



**Superradiant Phase** $g > \frac{\sqrt{\omega_c \omega_q}}{2}$

Double-well Potential



Superposition



# Introduction



**Finite-component quantum phase transitions**

~~$N \to \infty$~~ $N \sim 1$

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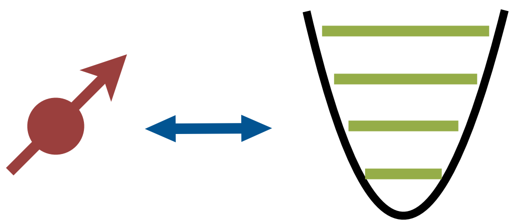
**Quantum Rabi model**



$$H_{\text{Rabi}} = \omega_0 a^\dagger a + \frac{\Omega}{2}\sigma_z - \lambda(a + a^\dagger)\sigma_x$$

Superradiant phase transition in the **scaling limit**

$$\Omega/\omega_0 \to \infty$$

L. Bakemeier, et al., Phys. Rev. A 85, 043821 (2012). M.-J. Hwang, et al., Phys. Rev. Lett. 115, 180404 (2015).

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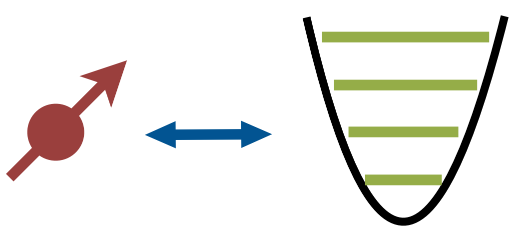

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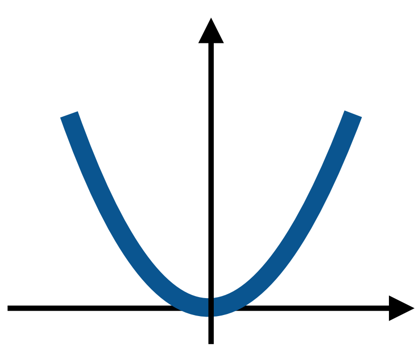

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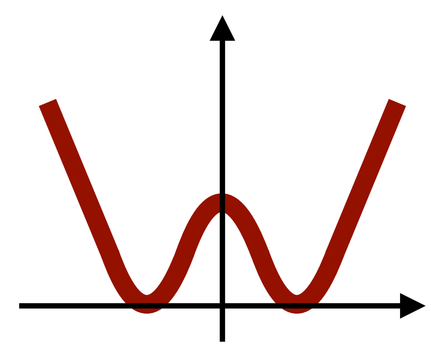

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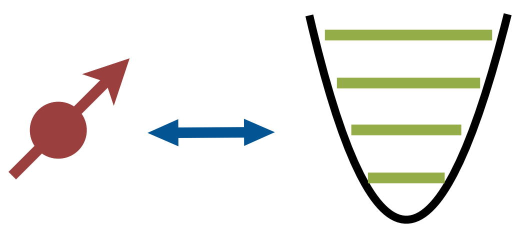

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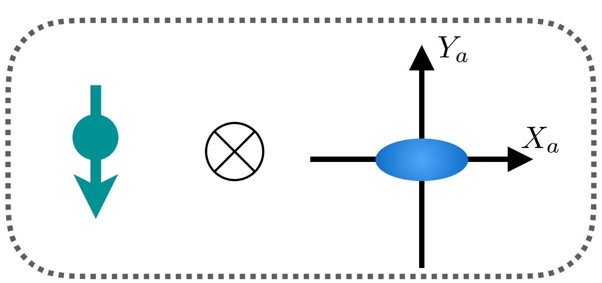
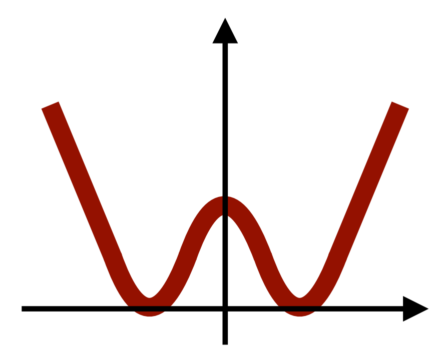



$$|\downarrow\rangle \otimes \mathcal{S}_\lambda |0\rangle$$

Superradiant phase $\lambda > \lambda_c$



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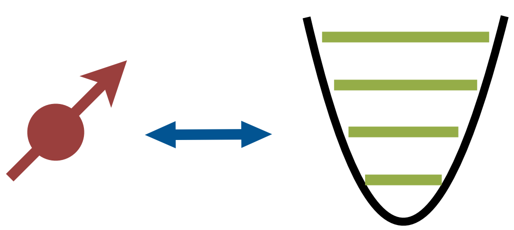

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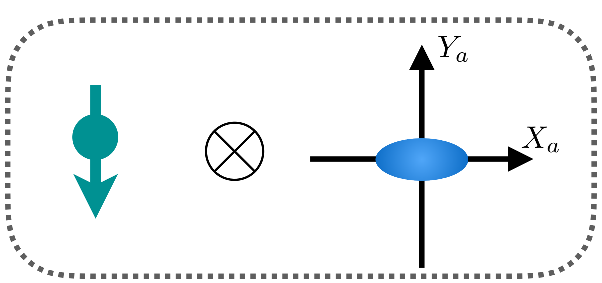


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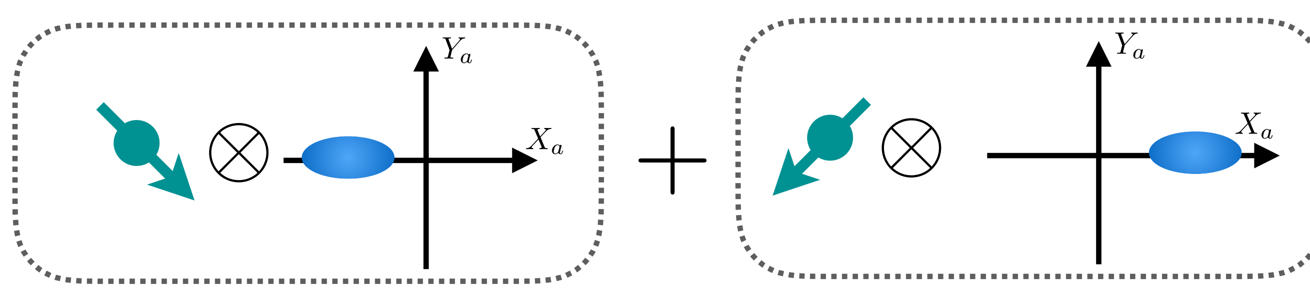
Superradiant phase $\lambda > \lambda_c$




$$|\swarrow\rangle \otimes \mathcal{D}_{-\alpha}\mathcal{S}_\chi |0\rangle + |\searrow\rangle \otimes \mathcal{D}_{\alpha}\mathcal{S}_\chi |0\rangle$$

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UAM Universidad Autónoma de Madrid

IFIMAC Condensed Matter Physics Center

erc

MPQ

Université de Paris

Louis Garbe

Arne Keller

Matteo Bina

Matteo Paris

Louis Garbe

Arne Keller

Matteo Bina

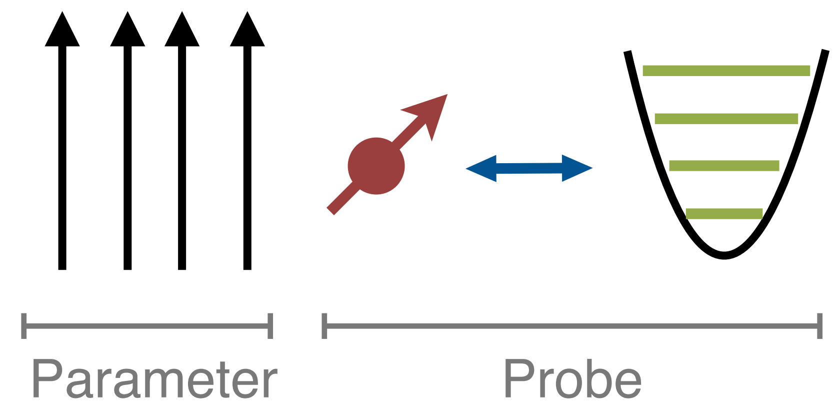
Matteo Paris

# Finite-component critical probe



$$H_{\text{Rabi}} = \omega_0 a^\dagger a + \frac{\Omega}{2}\sigma_z - \lambda(a + a^\dagger)\sigma_x$$

Scaling limit $\Omega/\omega_0 \gg 1$



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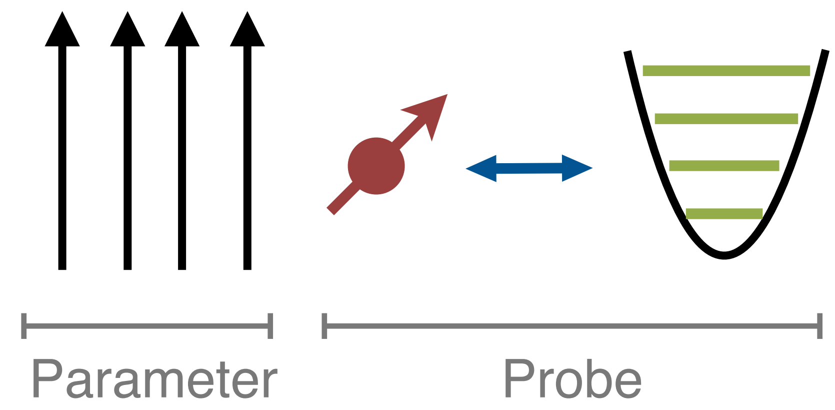


$$g = \lambda/\sqrt{\Omega\omega_0}$$

Quantum phase transition

$$g \to 1$$

# Finite-component critical probe



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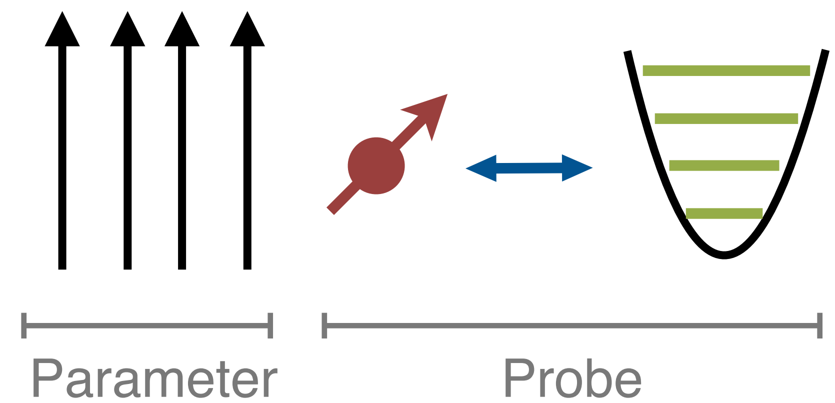


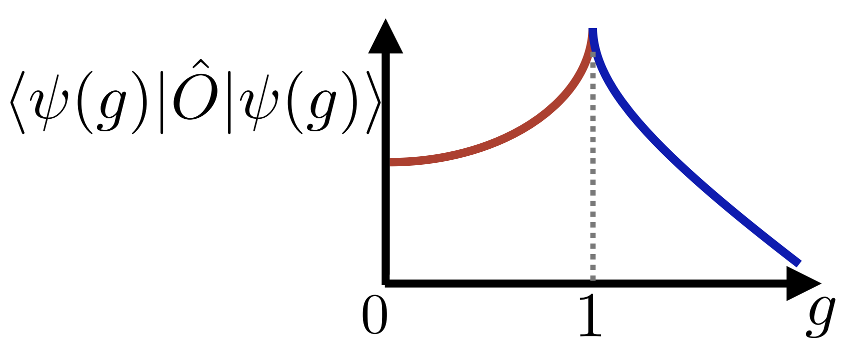

$$g = \lambda/\sqrt{\Omega\omega_0}$$

Quantum phase transition $g \to 1$

**Estimation protocol**

**1)** Prepare trivial ground state

$$|\psi(g)\rangle = |0\rangle \otimes |\downarrow\rangle$$



# Finite-component critical probe



$$H_{\mathrm{Rabi}} = \omega_0 a^\dagger a + \frac{\Omega}{2}\sigma_z - \lambda(a + a^\dagger)\sigma_x$$

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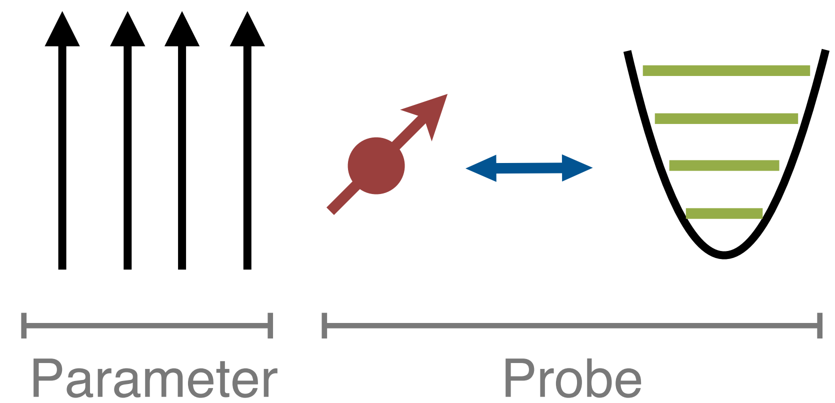


$$g = \lambda/\sqrt{\Omega\omega_0}$$ → Quantum phase transition $g \to 1$

**Estimation protocol**

**1)** Prepare trivial ground state

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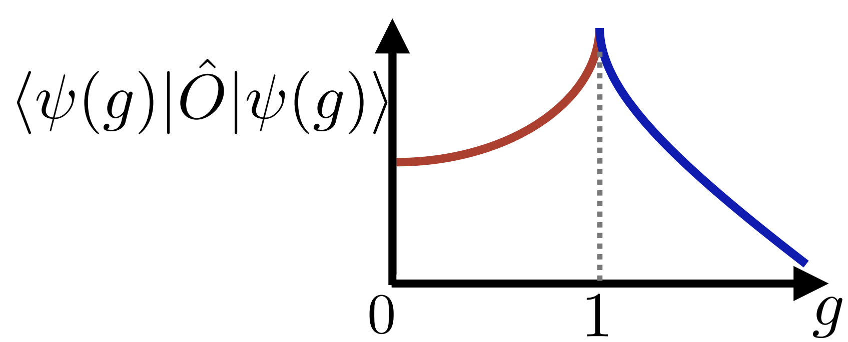
**2)** Adiabatic sweep

$$g(t) : 0 \longrightarrow 1$$



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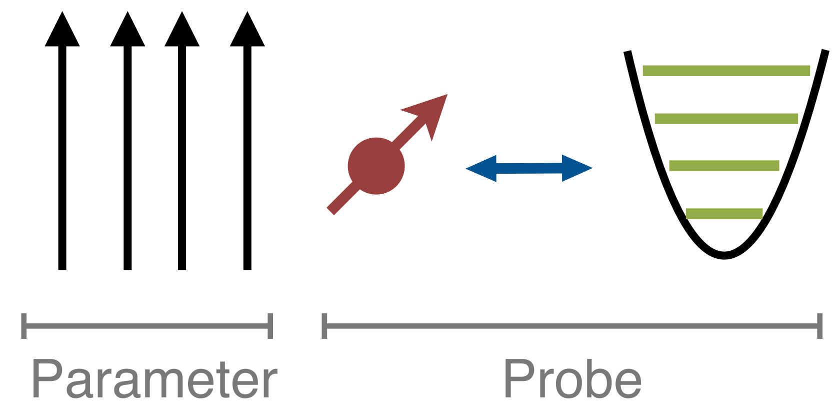



$$g = \lambda/\sqrt{\Omega\omega_0}$$

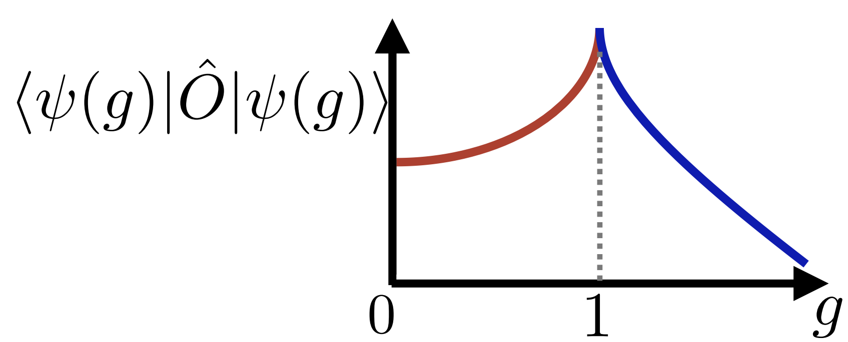
Quantum phase transition $g \to 1$

**Estimation protocol**

**1)** Prepare trivial ground state $|\psi(g)\rangle = |0\rangle \otimes |\downarrow\rangle$

**2)** Adiabatic sweep $g(t) : 0 \longrightarrow 1$

**3)** Measure ground state $\langle\psi(g)|\hat{O}|\psi(g)\rangle$

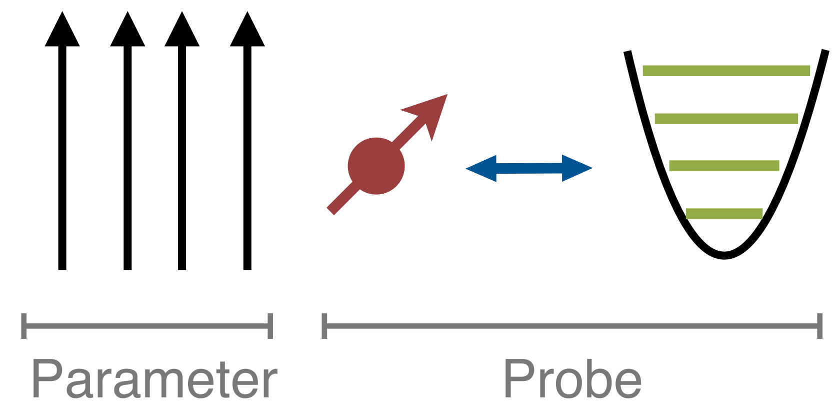


# Finite-component critical probe



$$H_{\text{Rabi}} = \omega_0 a^\dagger a + \frac{\Omega}{2}\sigma_z - \lambda(a + a^\dagger)\sigma_x$$

Scaling limit $\Omega/\omega_0 \gg 1$




$$g = \lambda/\sqrt{\Omega\omega_0}$$ → Quantum phase transition $g \to 1$

**Estimation protocol**

**1)** Prepare trivial ground state $|\psi(g)\rangle = |0\rangle \otimes |\downarrow\rangle$

**2)** Adiabatic sweep $g(t) : 0 \longrightarrow 1$

**3)** Measure ground state $\langle\psi(g)|\hat{O}|\psi(g)\rangle$

**Driven-dissipative case**

**1)** Prepare trivial initial state $|\psi(g)\rangle = |0\rangle \otimes |\downarrow\rangle$

**2)** Long-time evolution $\lim_{t\to\infty} |\psi(t)\rangle$

**3)** Measure steady state $\langle\psi(t)|\hat{O}|\psi(t)\rangle$

# Evaluation



## Quantum Fisher information

Ground state $|\psi(g)\rangle = \hat{S}(\xi)|0\rangle \otimes |\downarrow\rangle$

Squeezing $\xi = -\frac{1}{4}\log(1 - g^2)$

Q.F.I. $G_A = 4[\langle\partial_A\psi|\partial_A\psi\rangle + (\langle\partial_A\psi|\psi\rangle)^2]$

## Protocol duration

# Evaluation



## Quantum Fisher information

Ground state $|\psi(g)\rangle = \hat{S}(\xi)|0\rangle \otimes |{\downarrow}\rangle$

Squeezing $\xi = -\frac{1}{4}\log(1-g^2)$

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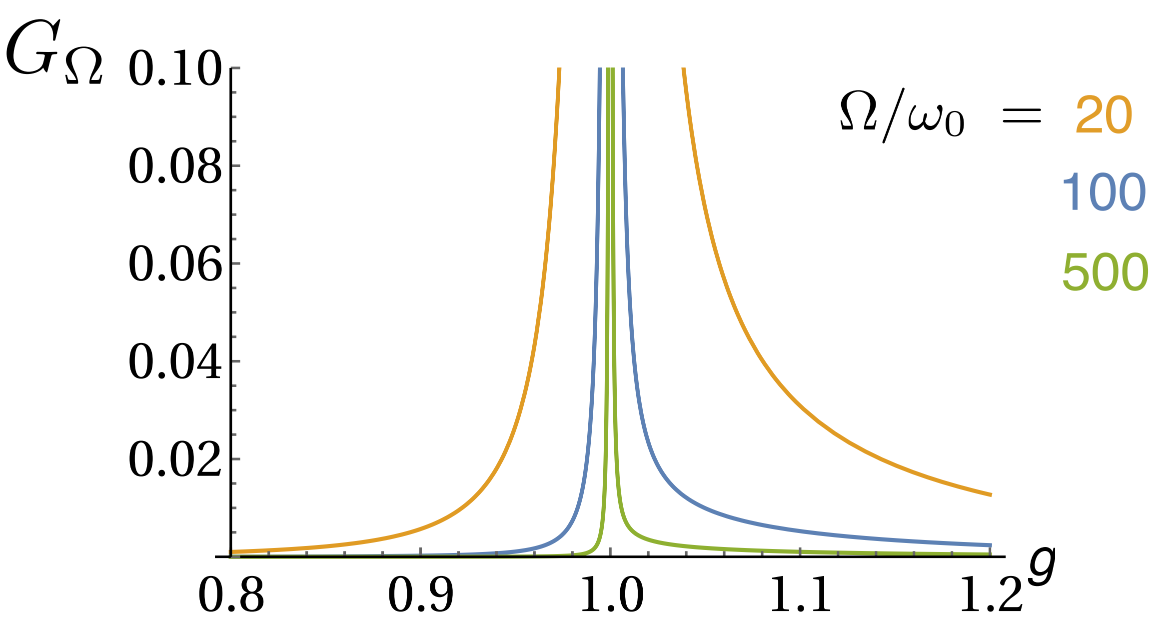


**Critical scaling Q.F.I.** $G_A \simeq \frac{1}{32\,A^2(1-g)^2}$

## Protocol duration

# Evaluation



## Quantum Fisher information

Ground state $|\psi(g)\rangle = \hat{S}(\xi)|0\rangle \otimes |\downarrow\rangle$

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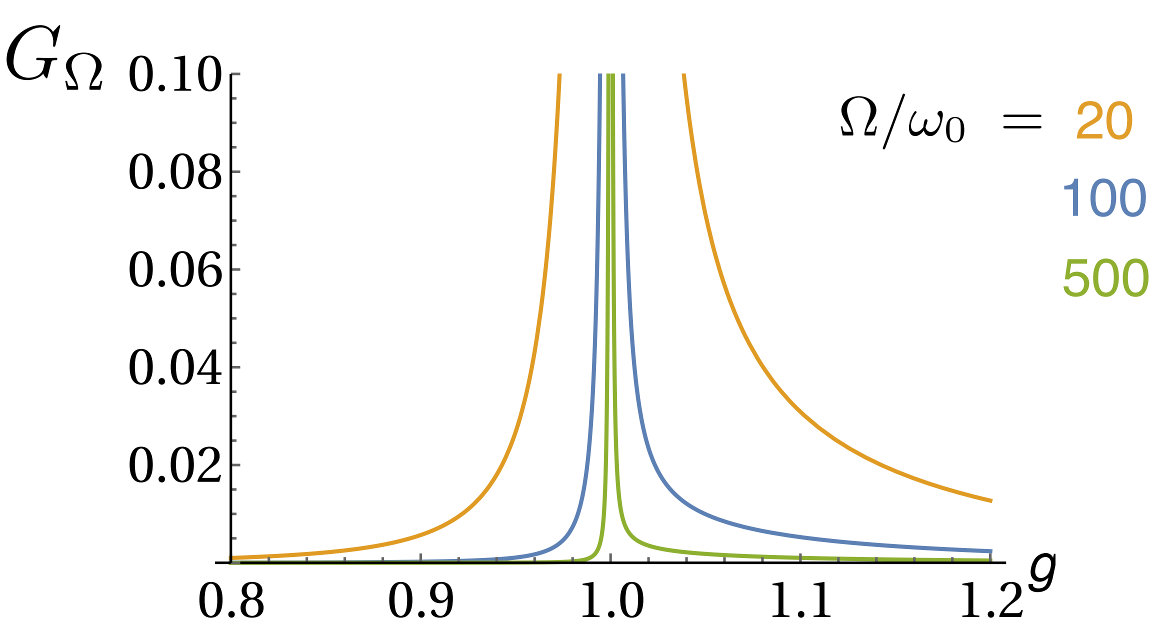
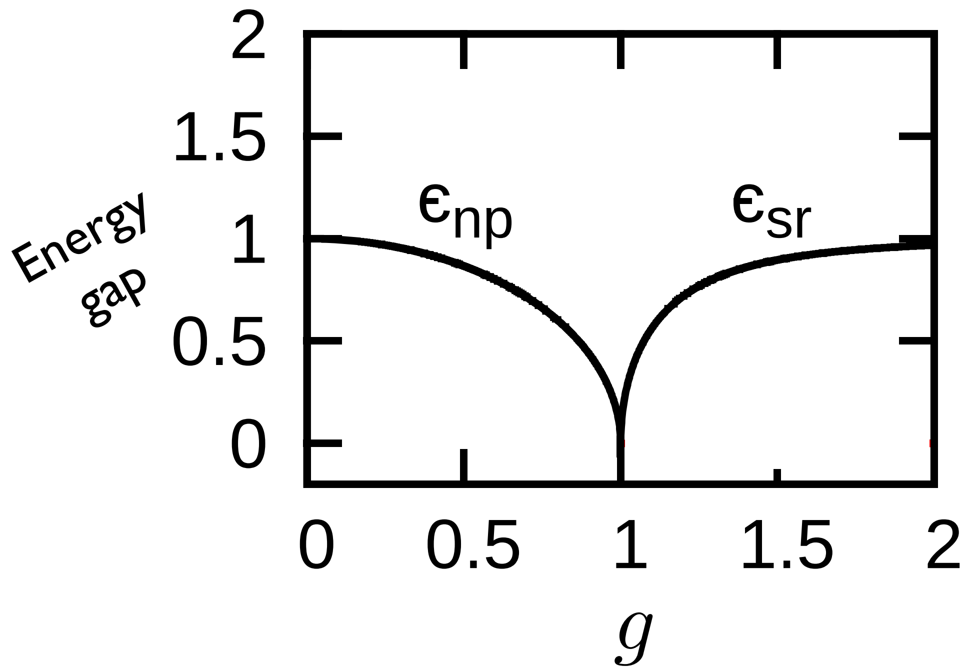



**Critical scaling Q.F.I.**

$$G_A \simeq \frac{1}{32\,A^2(1-g)^2}$$

## Protocol duration

$$\epsilon_{\rm np} = \omega_0\sqrt{1-g^2}$$



# Evaluation



## Quantum Fisher information

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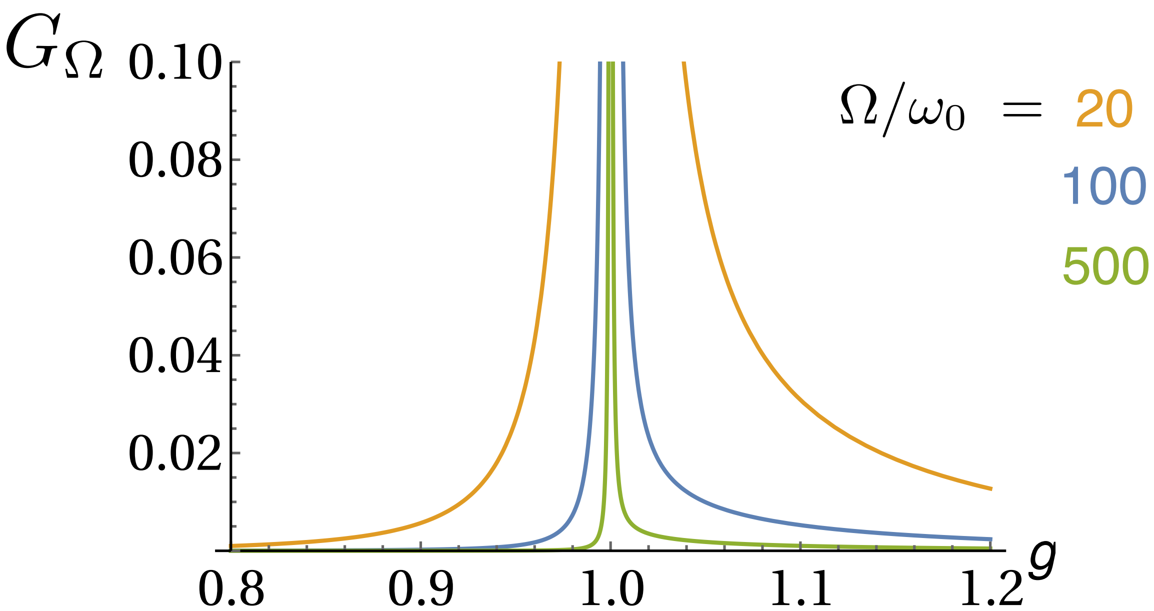


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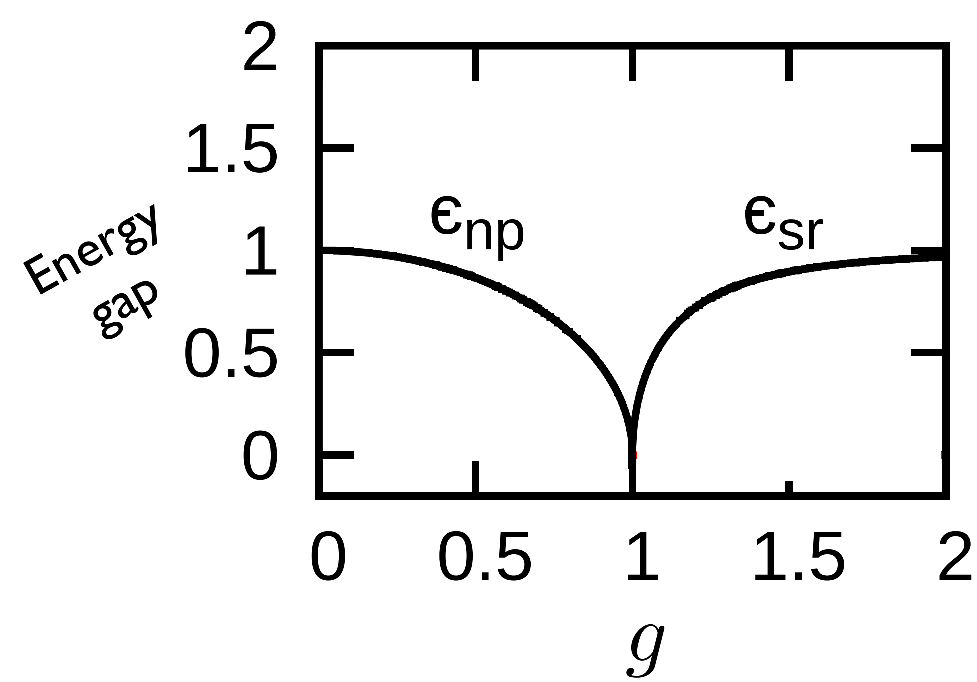
## Protocol duration

$\epsilon_{np} = \omega_0\sqrt{1-g^2}$




(from time-dependent perturbation theory)

Adiabatic evolution $v(g) \ll \frac{2g}{1+g^2}\,\omega_0\,(1-g^2)^{3/2}$

**Critical scaling Evolution time** $T = \int_0^g \frac{ds}{v(s)} \sim \frac{1}{\omega_0\sqrt{1-g}}$

# Results



**Analysis of the scaling of the estimation precision** $G_{\omega_0}$

Probe number $\langle \hat{N} \rangle$ Time

**Hamiltonian:** $G_{\omega_0} \sim \langle \hat{N} \rangle^2 T^2$ Saturate Heisenberg limit

**Driven-dissipative:** $G_{\omega_0} \sim \langle \hat{N} \rangle T$ Optimal in noisy Q. Metrology

# Results



**Analysis of the scaling of the estimation precision** $G_{\omega_0}$

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**Driven-dissipative:** $G_{\omega_0} \sim \langle \hat{N} \rangle T$ Optimal in noisy Q. Metrology

**Optimal scaling in spite of the critical slowing down!**

- L. Garbe, M. Bina, A. Keller, M. G. A. Paris, and S. Felicetti, Phys. Rev. Lett. **124**, 120504 (2020).

# Take-home message



**Finite-component phase transitions** ⟷ **Quantum metrology**

- L. Garbe, M. Bina, A. Keller, M. G. A. Paris, and S. Felicetti, Phys. Rev. Lett. **124**, 120504 (2020).

# Take-home message



**Finite-component phase transitions** ↔ **Quantum metrology**

$N \gg 1$ → $\Omega/\omega_0 \gg 1$

Termodynamic limit → Scaling limit

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# Take-home message



**Finite-component phase transitions** ⟷ **Quantum metrology**

$$N \gg 1$$

Termodynamic limit

→

$$\Omega/\omega_0 \gg 1$$

Scaling limit

**Fundamental interest**

Ultimate Precision limits

$\langle \hat{N} \rangle$

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# Take-home message



**Finite-component phase transitions** ⟷ **Quantum metrology**

$N \gg 1$

Termodynamic limit

→

$\Omega/\omega_0 \gg 1$

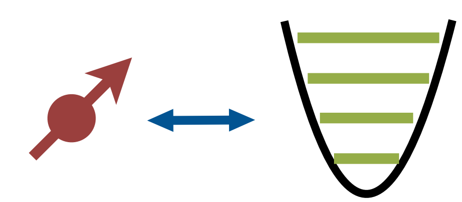
Scaling limit

**Fundamental interest**

Ultimate Precision limits $\langle \hat{N} \rangle$

**Practical consequences**

Few-body critical probe



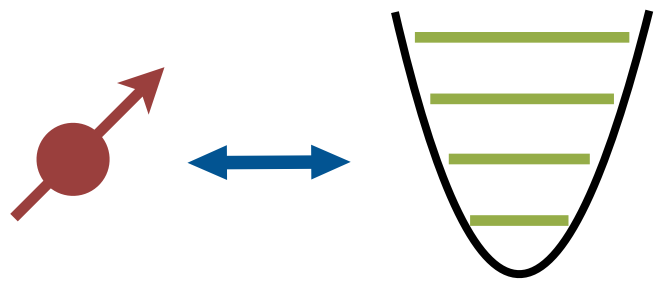
- L. Garbe, M. Bina, A. Keller, M. G. A. Paris, and S. Felicetti, Phys. Rev. Lett. **124**, 120504 (2020).

# Finite-component phase transitions

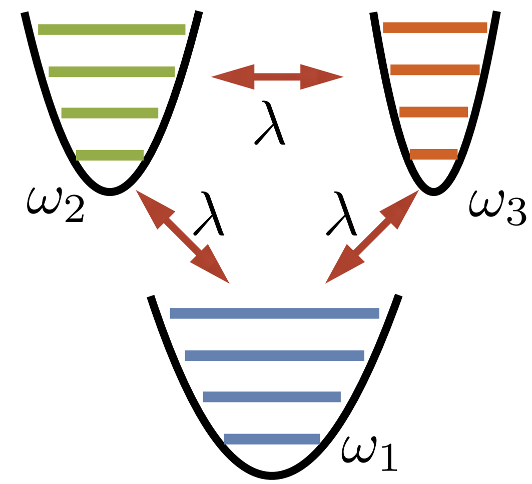


## A universal feature of ultrastrong coupling

**Quantum Rabi model**



**Nonlinear quantum resonators**

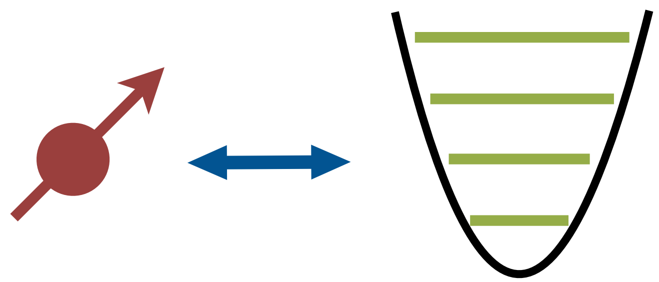



- S. Felicetti and A. Le Boité, Phys. Rev. Lett. **124**, 040404 (2020).

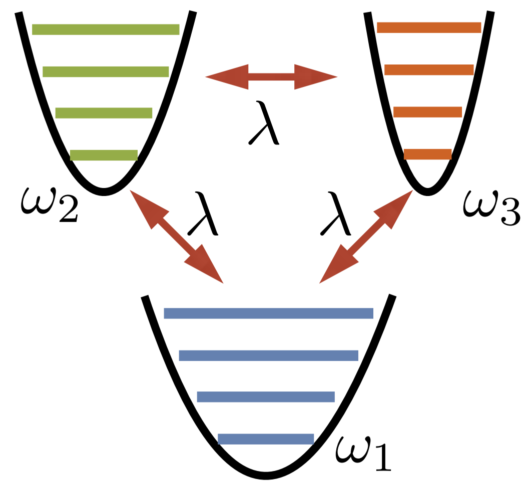
# Finite-component phase transitions

## A universal feature of ultrastrong coupling

### Quantum Rabi model



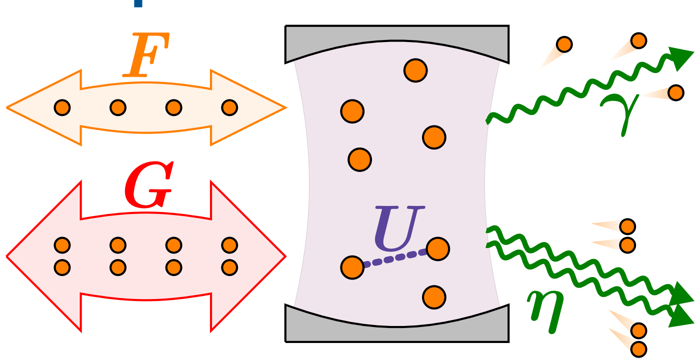
### Nonlinear quantum resonators




- S. Felicetti and A. Le Boité, Phys. Rev. Lett. **124**, 040404 (2020).

## Take place in driven-dissipative systems

### Pumped Kerr resonators



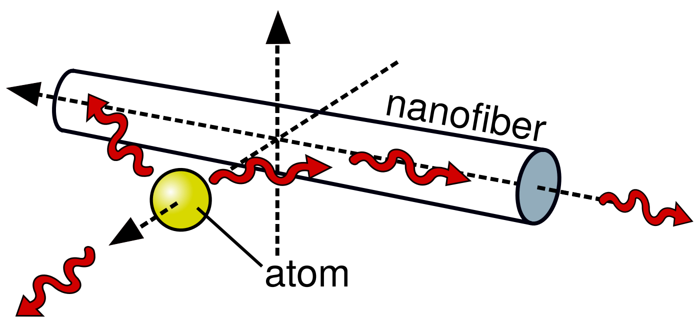

- N. Bartolo, F. Minganti, W. Casteels, and C. Ciuti, Phys. Rev. A **94**, 033841 (2016).

- R. Rota, F. Minganti, C. Ciuti, and V. Savona, Phys. Rev. A **112**, 110405 (2019).

# Finite-component phase transitions

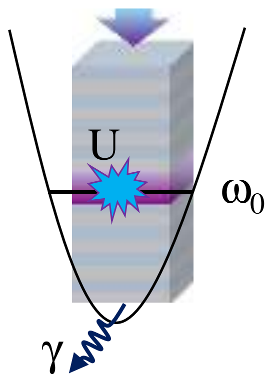
## Implementations in quantum technologies

### Atomic Systems



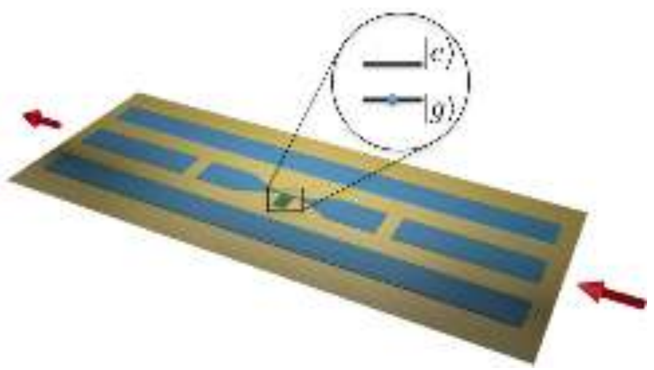

- A. Dareau et al., PRL **121**, 253603 (2018).
- M.-L. Cai et al., Nat. Comm. **12**, 1126 (2021).

### Polaritonics



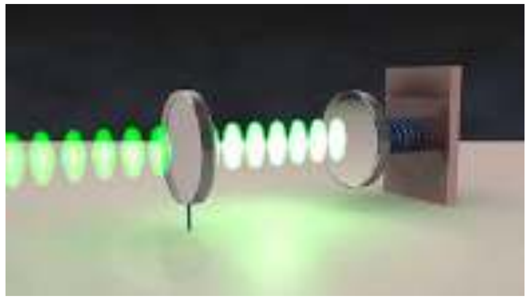

- S. R. K. Rodriguez et al., PRL 118, 247402 (2017).
- T. Fink et al., Nat. Phys 14, 365 (2018).

### Circuit QED



- D. Marcovic et al., PRL **121**, 040505 (2018).

### Opto/electro-mechanics



- G. Peterson et al., PRL **123**, 247701(2019).

Aalto University

RIKEN

CNR IFN Istituto di Fotonica e Nanotecnologie

Roberto
di Candia

Fabrizio
Minganti

Kirill
Petrovnin

G. Sorin
Paraoanu

- R. Di Candia *, F. Minganti *, K.V. Petrovnin, G. S. Paraoanu, and S. Felicetti, arXiv:2107.04503 (2021).

# Parametric quantum sensor

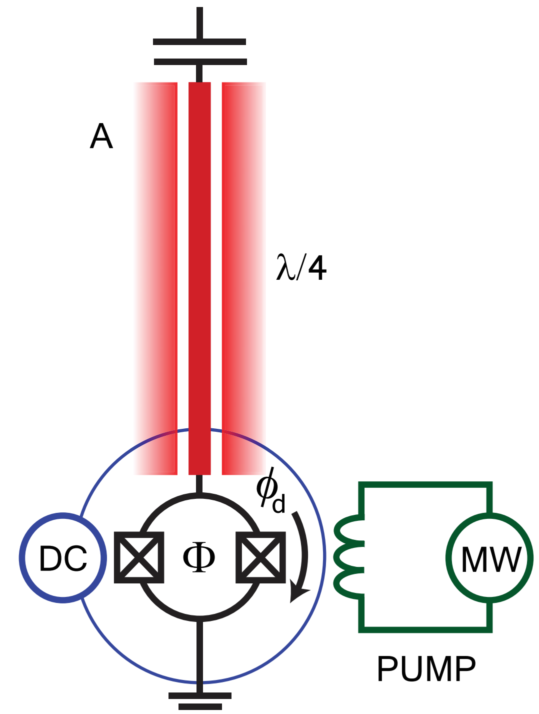


**Master equation**

$$\dot{\rho} = -i[H, \rho] + \kappa N(a^\dagger \rho a - 1/2\left\{aa^\dagger, \rho\right\})$$

**Hamiltonian**

$$\hat{H}_{\text{Kerr}}/\hbar = \omega \hat{a}^\dagger \hat{a} + \frac{\epsilon}{2}(\hat{a}^{\dagger 2} + \hat{a}^2) + \chi \hat{a}^{\dagger 2} \hat{a}^2$$




- P. Krantz et al, New J. Phys. **15** 105002 (2013).

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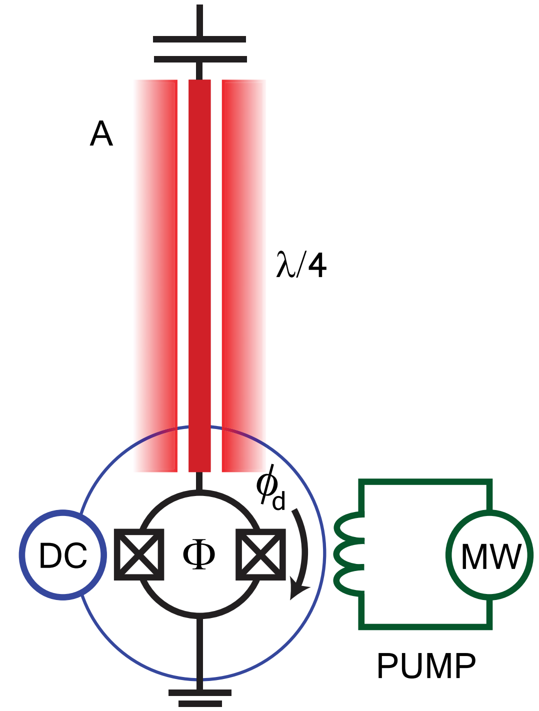
**Master equation**

$$\dot{\rho} = -i[H,\rho] + \kappa N(a^\dagger \rho a - 1/2\left\{aa^\dagger,\rho\right\})$$

**Hamiltonian**

$$\hat{H}_{\rm Kerr}/\hbar = \omega \hat{a}^\dagger \hat{a} + \frac{\epsilon}{2}(\hat{a}^{\dagger 2} + \hat{a}^2) + \chi \hat{a}^{\dagger 2}\hat{a}^2$$

**Weak (but finite!) nonlinearity**
$\chi \ll 1$
→
**Critical PT steady state**

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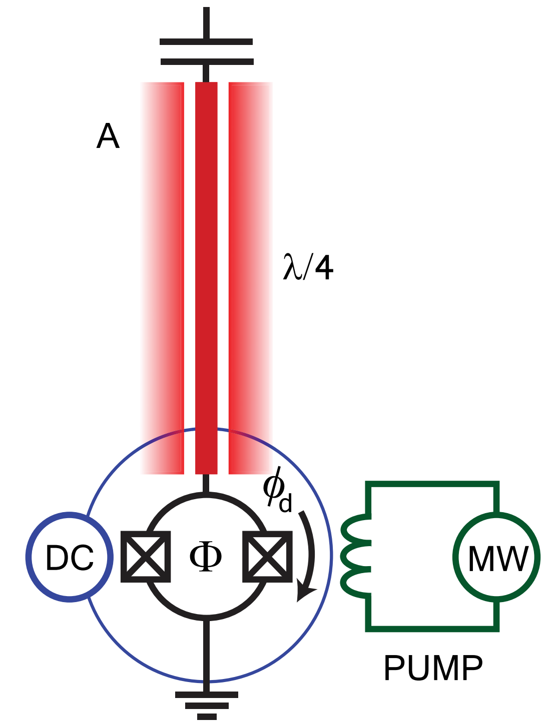


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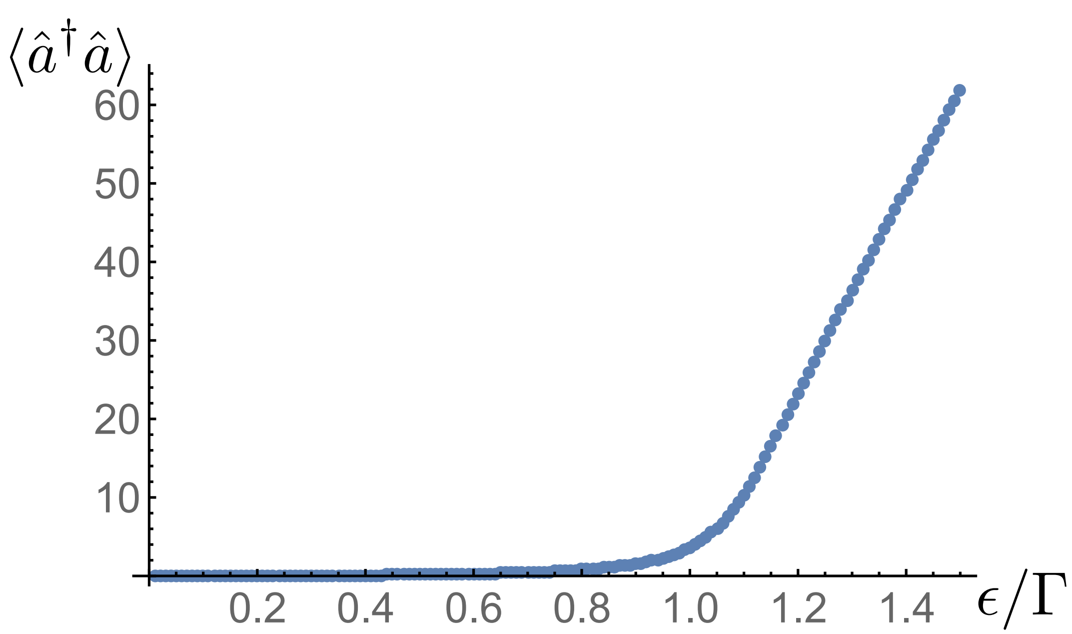
**Weak (but finite!) nonlinearity** $\chi \ll 1$ → **Critical PT steady state**

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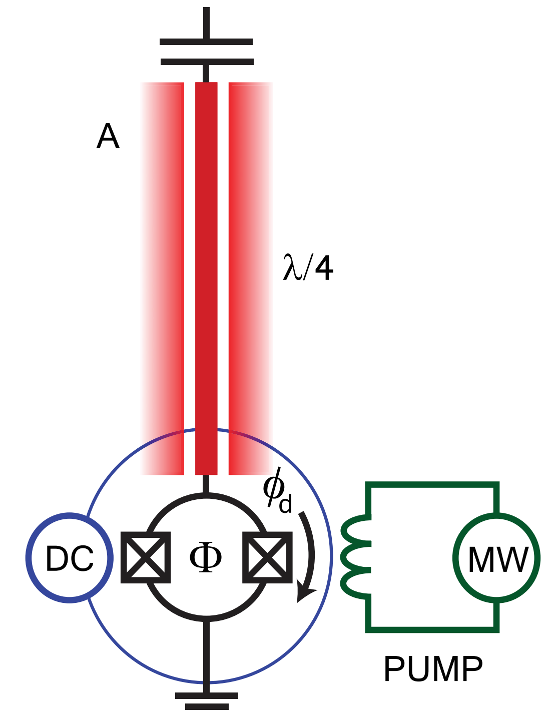


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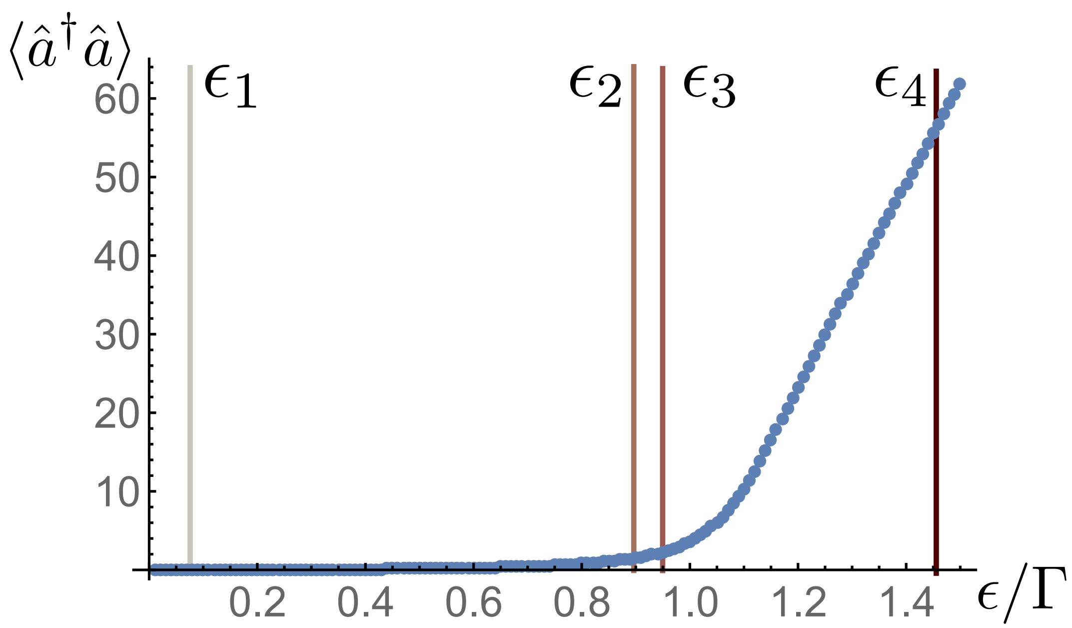


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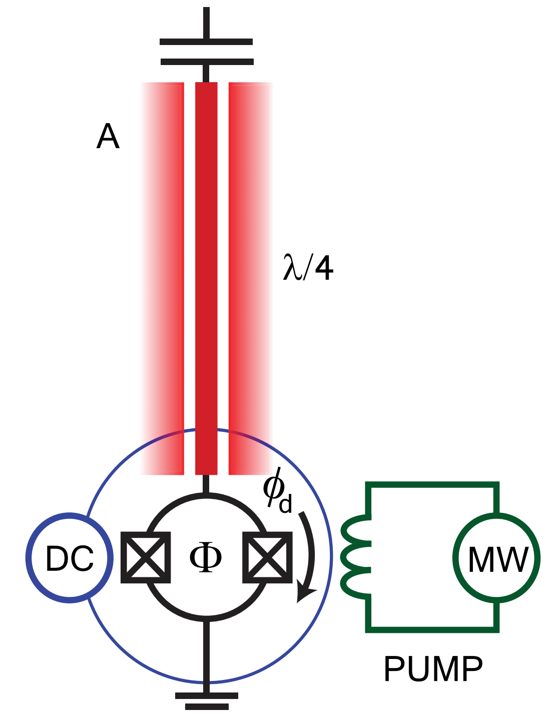


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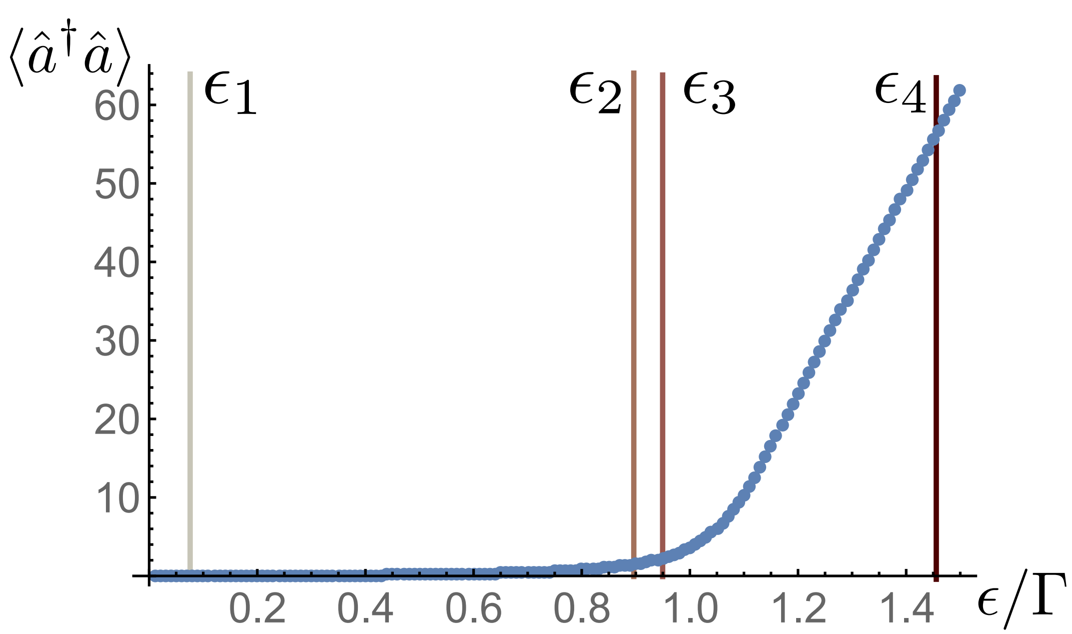


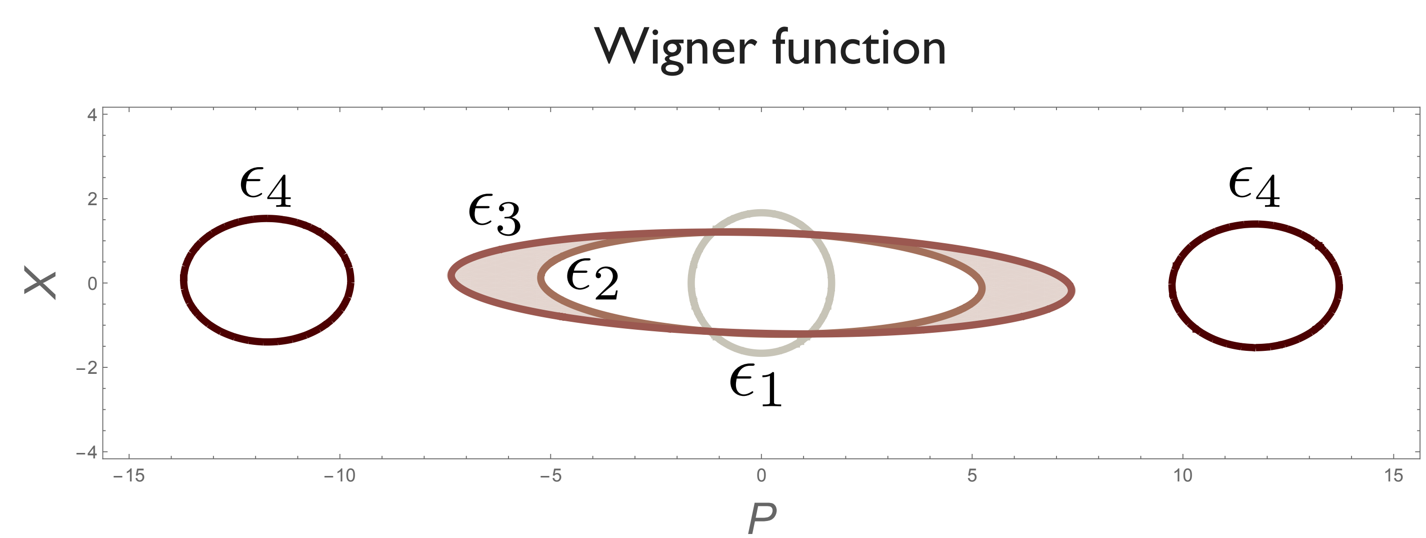


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# Magnetometry



**Master equation**

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**Hamiltonian**

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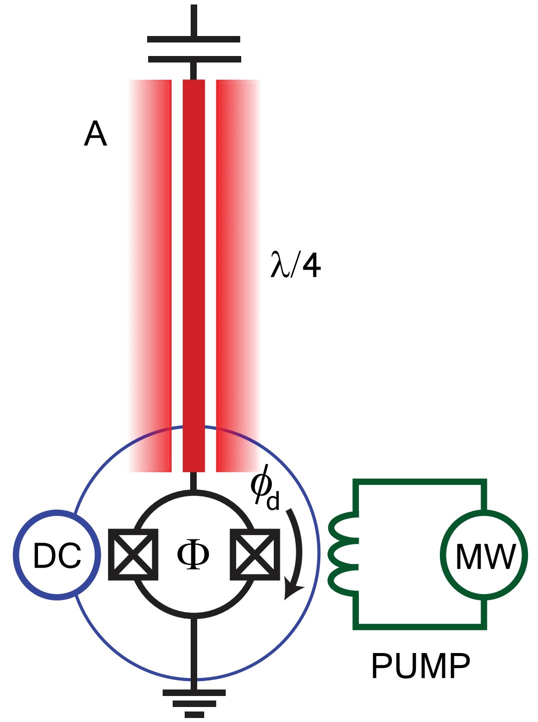
$$\omega = \omega(B)$$

**Magnetic field estimation**

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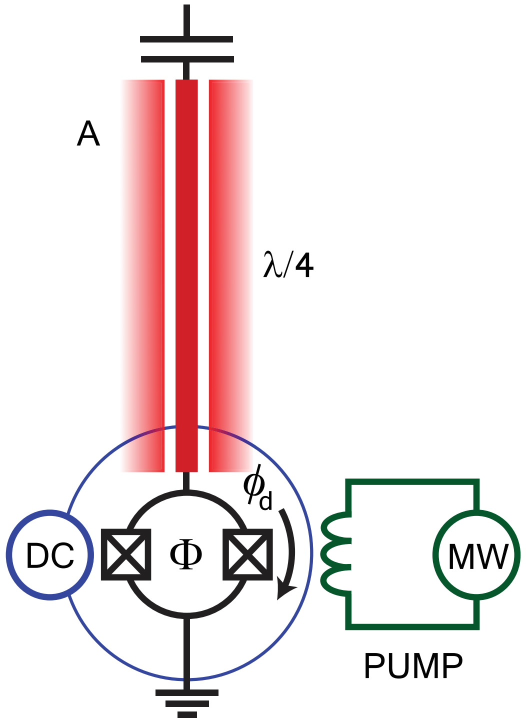


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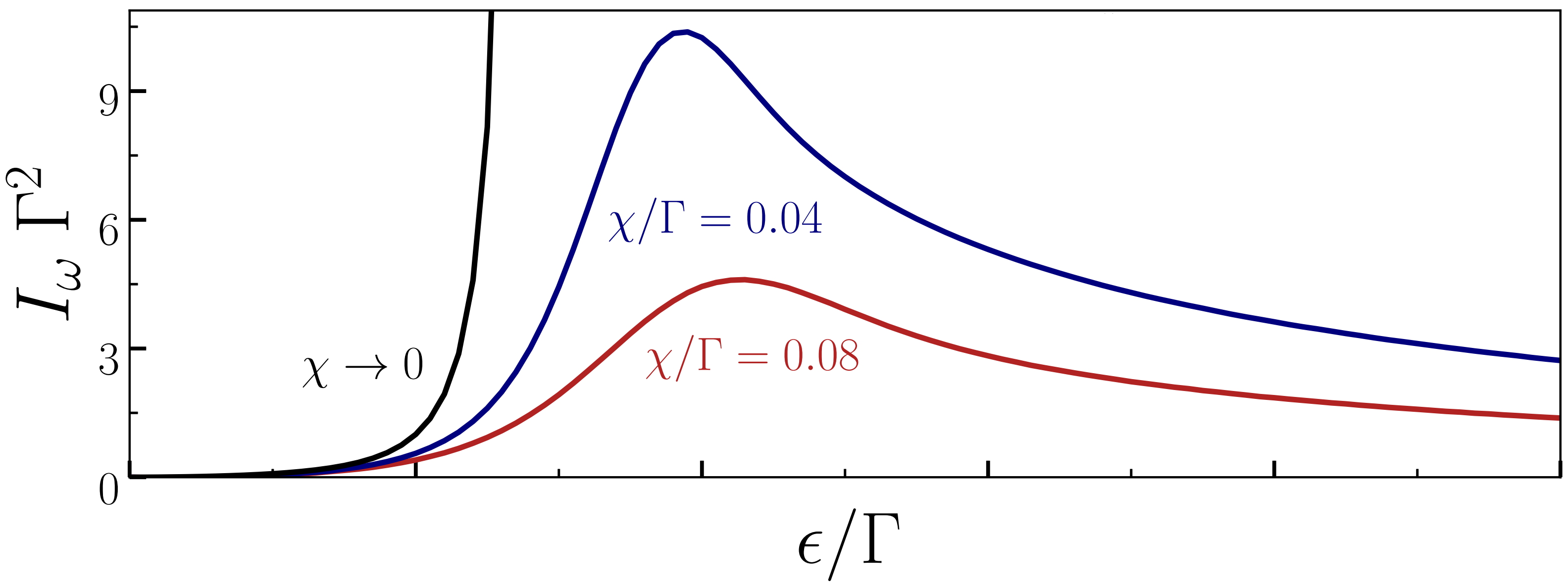
**Quantum Fisher information**



# Magnetometry



**Master equation**

$$\dot{\rho} = -i[H,\rho] + \kappa N(a^\dagger \rho a - 1/2\{aa^\dagger, \rho\})$$

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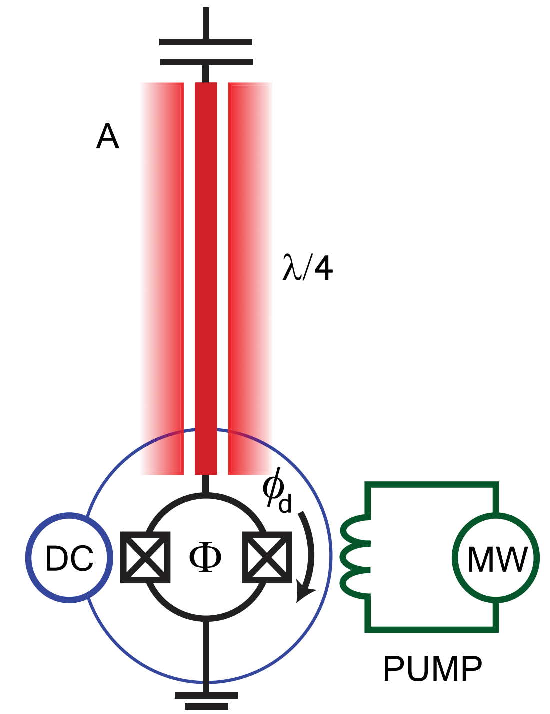


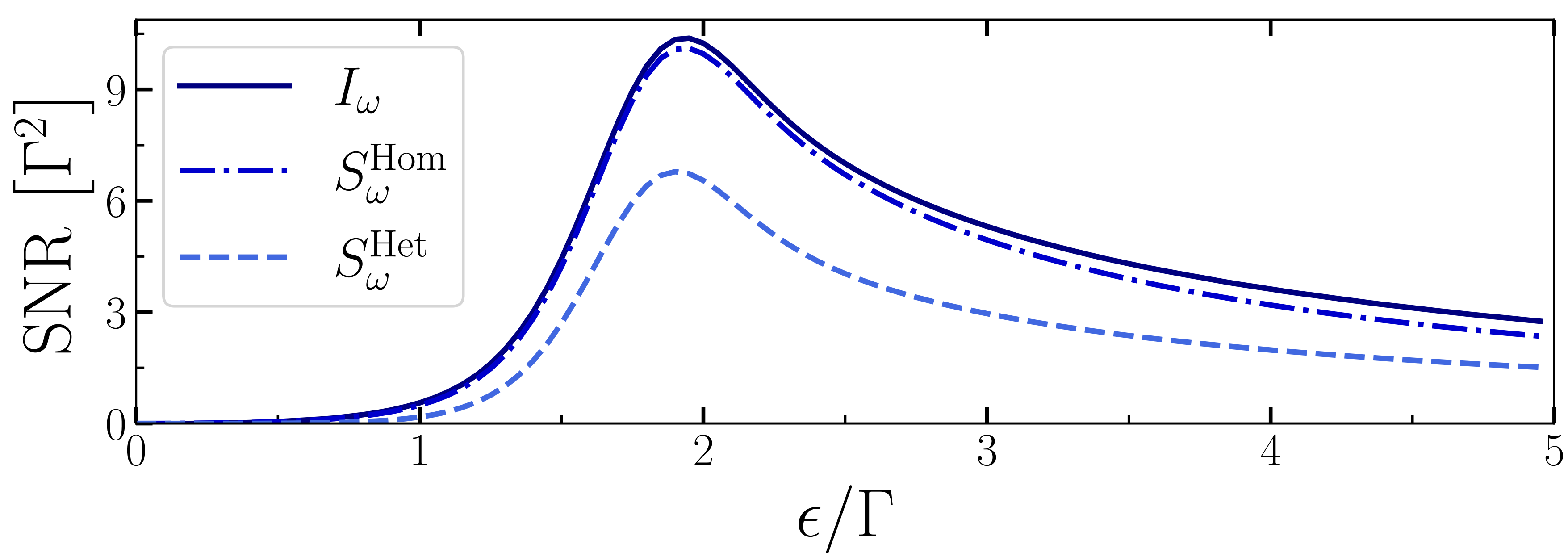

- P. Krantz et al, New J. Phys. **15** 105002 (2013).

**Signal-to-noise ratio**

Fix measurement

$$SNR_\omega(\hat{O}) = \frac{[\partial_\omega \langle \hat{O}\rangle_\omega]^2}{\Delta O_\omega^2}$$

$$\Delta O_\omega^2 = \langle \hat{O}^2\rangle_\omega - \langle \hat{O}\rangle_\omega^2$$



# Qubit readout

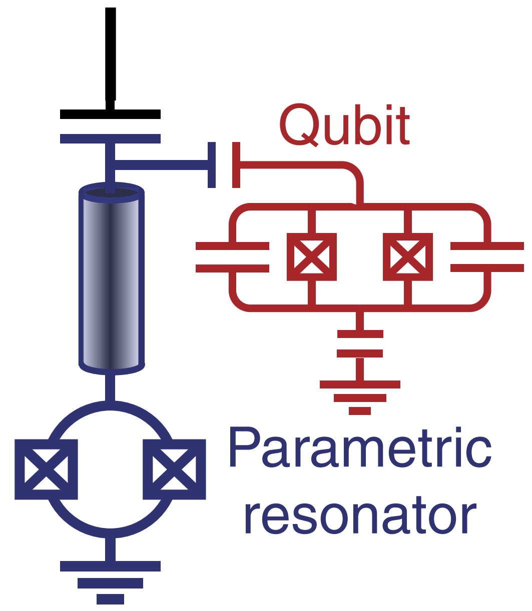


**Master equation**

$$\dot{\rho} = -i[H, \rho] + \kappa N(a^\dagger \rho a - 1/2 \{aa^\dagger, \rho\})$$

**Hamiltonian**

$$\hat{H}_{\mathrm{Kerr}}/\hbar = \omega \hat{a}^\dagger \hat{a} + \frac{\epsilon}{2}(\hat{a}^{\dagger 2} + \hat{a}^2) + \chi \hat{a}^{\dagger 2} \hat{a}^2$$

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# Qubit readout



**Master equation**

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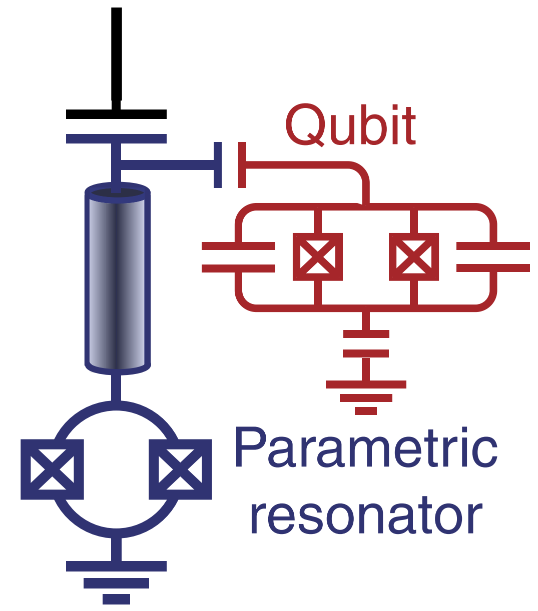
**Dispersive coupling** $H_{qc} = \delta\omega\ \hat{\sigma}_z \hat{a}^\dagger \hat{a}$

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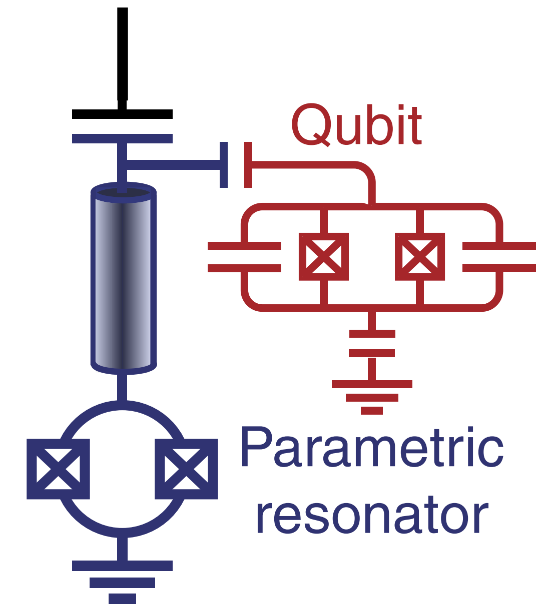



- P. Krantz et al, Nat. Comm. **7** 11417 (2016).

**State discrimination:**

$$|0\rangle \longrightarrow \omega' = \omega - \delta\omega$$

$$|1\rangle \longrightarrow \omega' = \omega + \delta\omega$$

# Qubit readout



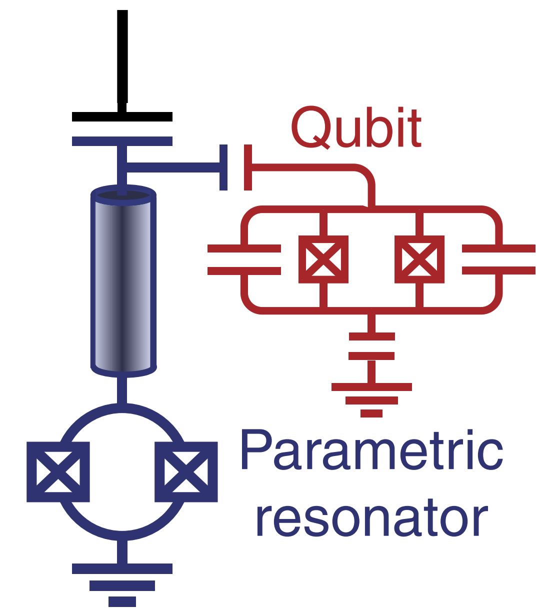
**Master equation**

$$\dot{\rho} = -i[H,\rho] + \kappa N(a^\dagger \rho a - 1/2\,\{aa^\dagger,\rho\})$$

**Hamiltonian**

$$\hat{H}_{\mathrm{Kerr}}/\hbar = \omega \hat{a}^\dagger \hat{a} + \frac{\epsilon}{2}(\hat{a}^{\dagger 2} + \hat{a}^2) + \chi \hat{a}^{\dagger 2}\hat{a}^2$$

**Dispersive coupling** $H_{qc} = \delta\omega\ \hat{\sigma}_z \hat{a}^\dagger \hat{a}$



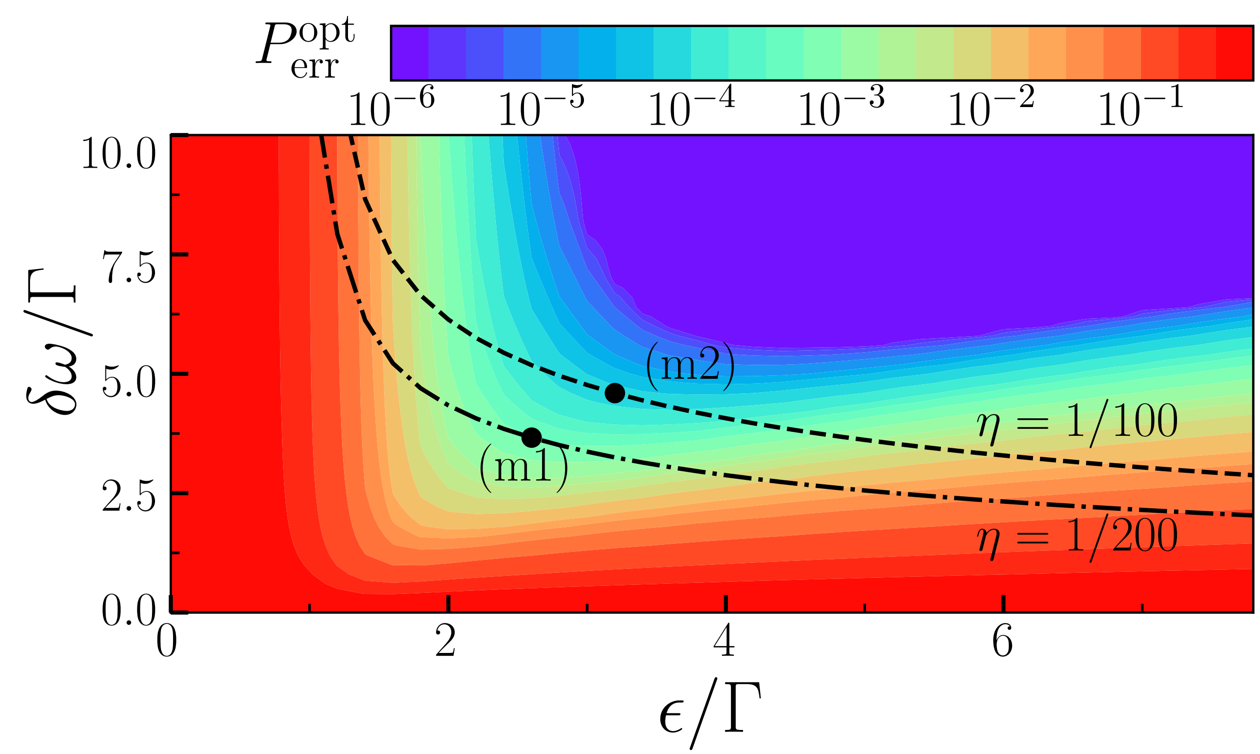

- P. Krantz et al, Nat. Comm. **7** 11417 (2016).

**State discrimination:**

$$|0\rangle \longrightarrow \omega' = \omega - \delta\omega$$

$$|1\rangle \longrightarrow \omega' = \omega + \delta\omega$$

**Readout error**



# Qubit readout

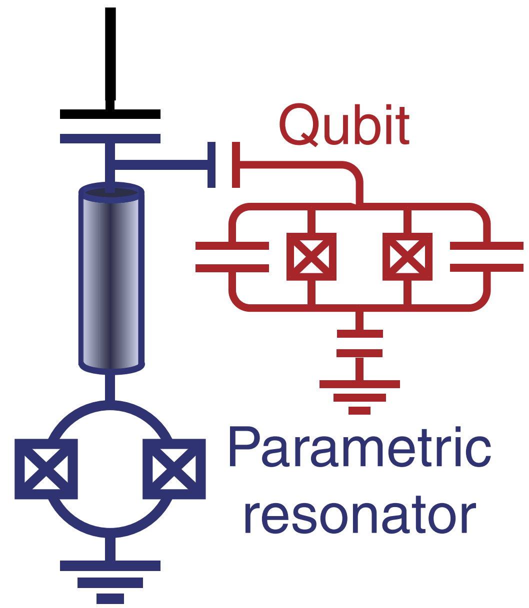


**Master equation**

$$\dot{\rho} = -i[H,\rho] + \kappa N(a^\dagger \rho a - 1/2\,\{aa^\dagger, \rho\})$$

**Hamiltonian**

$$\hat{H}_{\mathrm{Kerr}}/\hbar = \omega \hat{a}^\dagger \hat{a} + \frac{\epsilon}{2}(\hat{a}^{\dagger 2} + \hat{a}^2) + \chi \hat{a}^{\dagger 2}\hat{a}^2$$

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**State discrimination:**

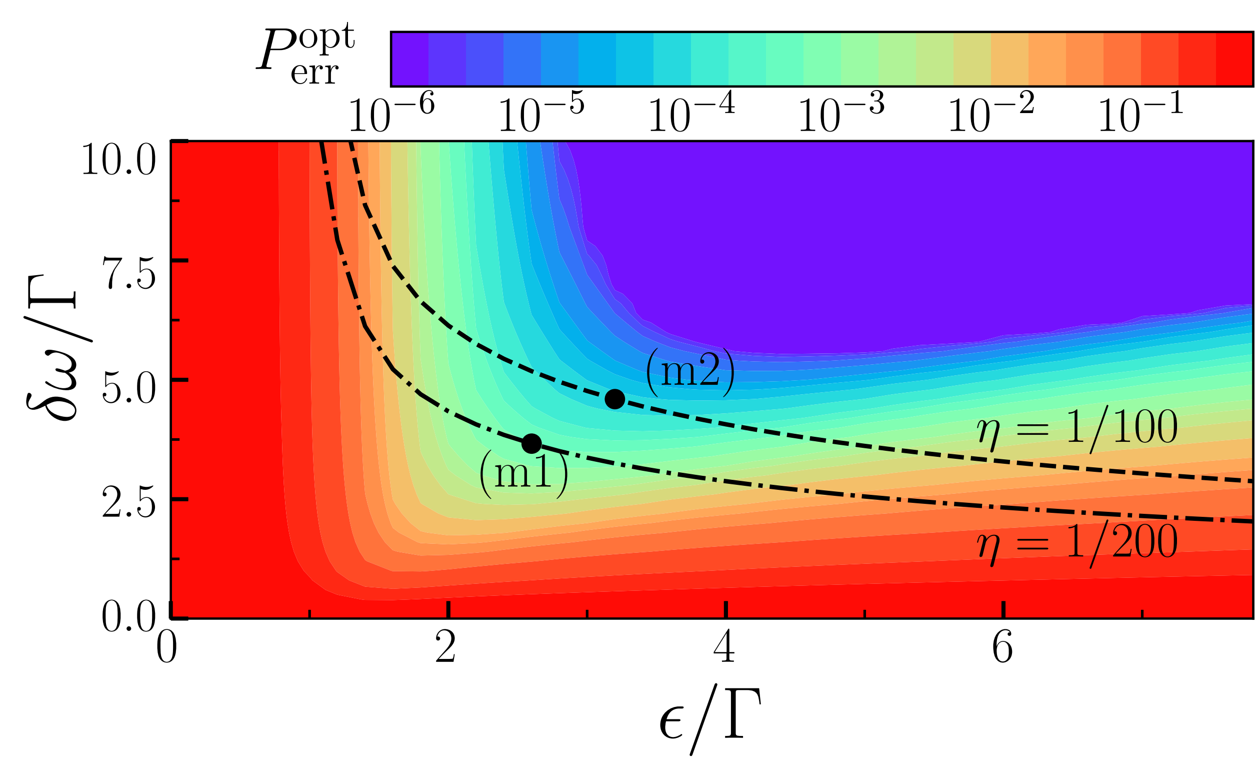
$$|0\rangle \longrightarrow \omega' = \omega - \delta\omega$$

$$|1\rangle \longrightarrow \omega' = \omega + \delta\omega$$

**Qubit degradation is proportional to:**

$$\eta = N\delta\omega^2/(4g^2)$$

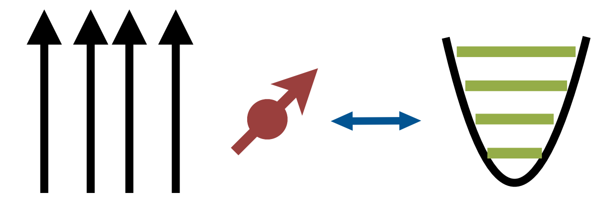
**Readout error**



# Conclusions



- **Finite-component PT for optimal quantum sensing**



- L. Garbe, M. Bina, A. Keller, M. G. A. Paris, and S. Felicetti, Phys. Rev. Lett. **124**, 120504 (2020).

# Conclusions



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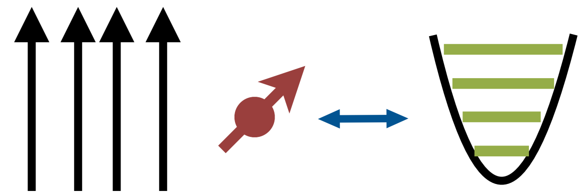
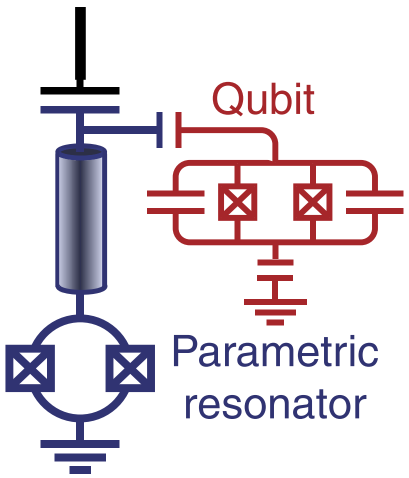


- L. Garbe, M. Bina, A. Keller, M. G. A. Paris, and S. Felicetti, Phys. Rev. Lett. **124**, 120504 (2020).

- **Critical parametric quantum sensor**

1- Quantum Magnetometry

2- Qubit readout




- R. Di Candia*, F. Minganti*, K.V. Petrovnin, G. S. Paraoanu, and S. Felicetti, arXiv:2107.04503 (2021).

# Conclusions



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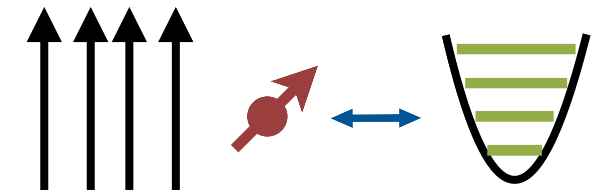

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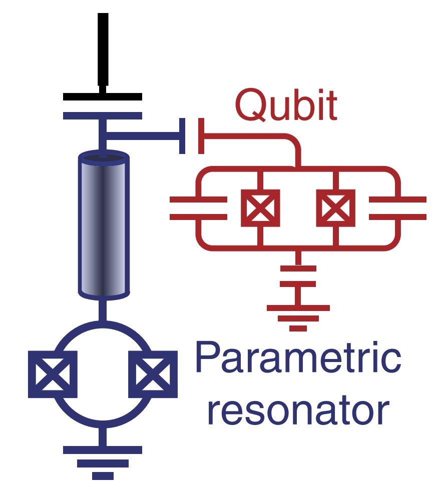



- R. Di Candia*, F. Minganti*, K.V. Petrovnin, G. S. Paraoanu, and S. Felicetti, arXiv:2107.04503 (2021).

## Recent works

**Sensing protocols**

- Y. Chu et al., PRL **126**, 010502 (2021).
- Gietka et al., arXiv:2103.12939 (2021).
- Y. Hu et al., arXiv:2101.01504 (2021).

**QPT phenomenology**

- H. Zhu et al. Phys. Rev. Lett. **125**, 050402 (2020).
- R. Puebla et al. Phys. Rev. B **102**, 220302(R) (2021).

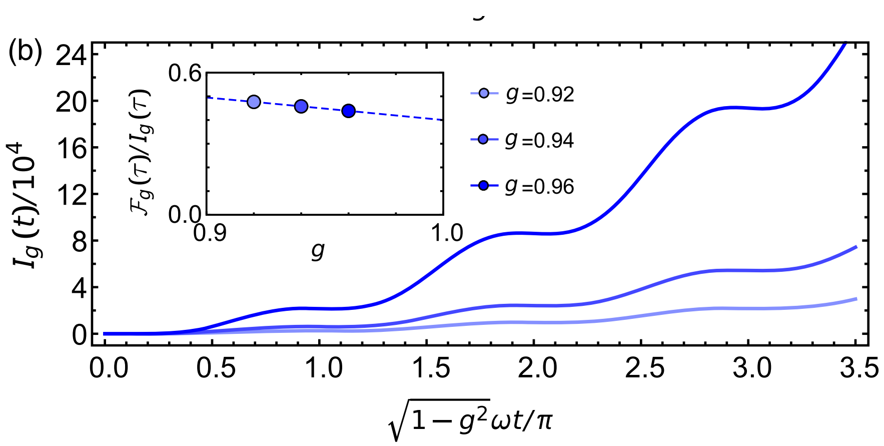
**Experiments**

- R. Liu et al., arXiv:2102.07056 (2021).

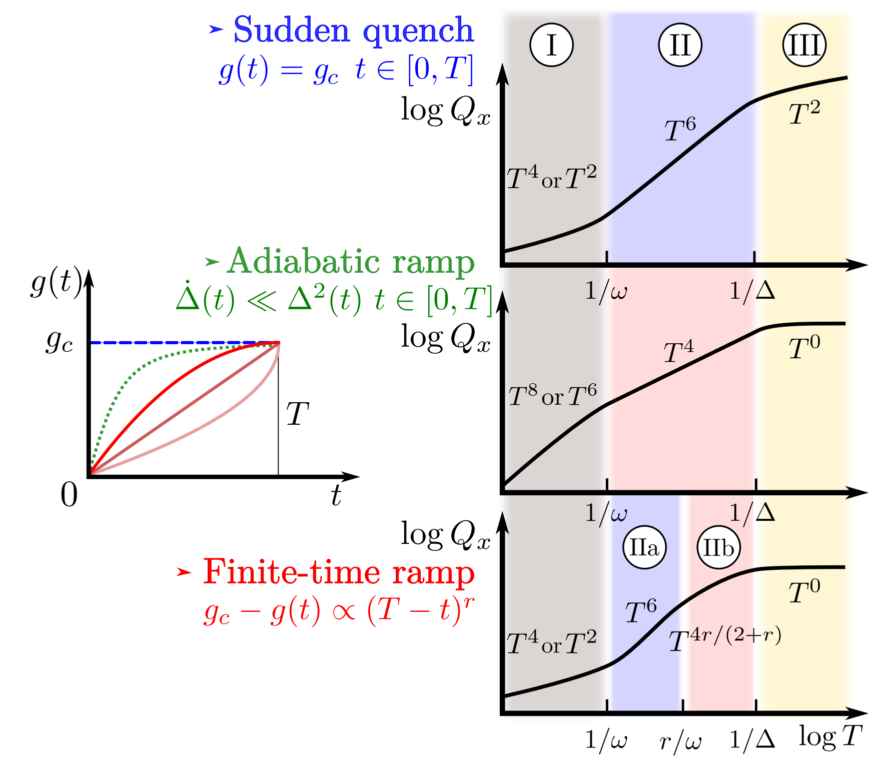
# Perspectives



## Dynamical protocols




- Y Chu, S Zhang, B Yu, J Cai, PRL **126**, 010502 (2021).

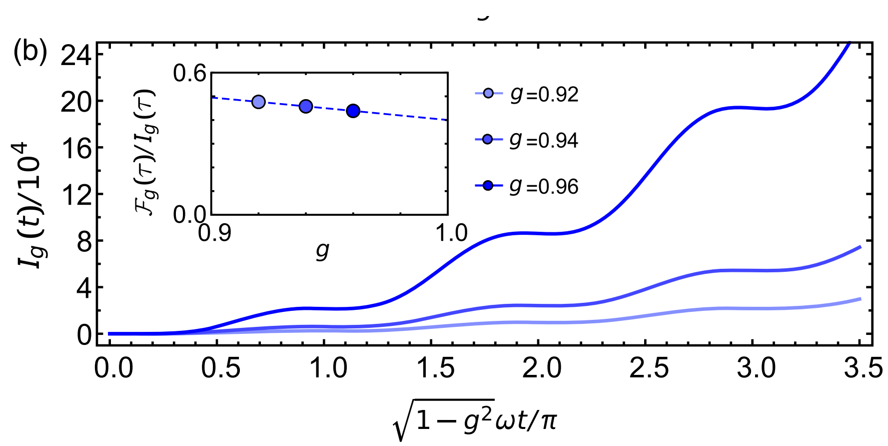



- L. Garbe, O. Abah, S. Felicetti, R. Puebla, arXiv:2110.04144 (2021).

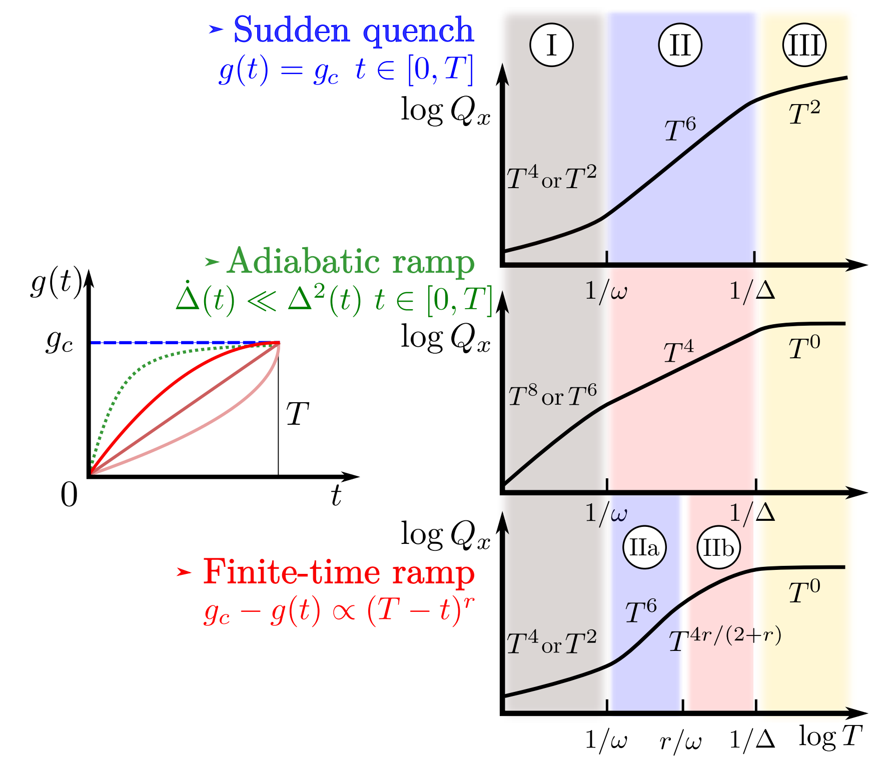
# Perspectives



## Dynamical protocols




- Y Chu, S Zhang, B Yu, J Cai, PRL **126**, 010502 (2021).




- L. Garbe, O. Abah, S. Felicetti, R. Puebla, arXiv:2110.04144 (2021).

## Open questions

- Can we overcome the critical slowing down?

- What is the impact of previous knowledge?